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Application of Nonlinear Superposition to Creep and Relaxation of Commercial Die-Casting Aluminum Alloys*

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Abstract. Die-cast aluminum alloys are heavily used in small engines, where they are subjected to long-term stresses at elevated temperatures. The resulting time-dependent material responses can result in inefficient engine operation and failure. A method to analytically determine the stress relaxation response directly from creep tests and to accurately interpolate between experimental time-history curves would be of great value. Constant strain, stress relaxation tests and constant load, creep tests were conducted on aluminum die-casting alloys: B-390, eutectic Al–Si and a 17% Si–Al alloy. A nonlinear superposition integral was used to (i) interpolate between empirical primary inelastic creep-strain and stress-relaxation time histories and (ii) to determine the stress relaxation response from corresponding creep data. Using isochronal stress-strain curves, prediction of the creep response at an intermediate stress level from empirical creep curves at higher and lower stresses resulted in a correlation (R) of 0.98. Similarly for relaxation, correlations of 0.98 were obtained for the prediction of an intermediate strain level curve from higher and lower empirical relaxation curves. The theoretical prediction of stress relaxation from empirical creep curves fell within 10% of experimental data.

Key words: aluminum, constitutive models, creep, interpolation, interrelation, nonlinear superposition, relaxation, viscoelasticity

1. Introduction

1.1. Need for Material Characterization and Modeling

Currently, small engine design is done with the designer's awareness of the time-dependent nature of their materials. Extensive data have been collected and can

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be found in the literature for creep in pure aluminum (Servi and Grant, 1951; Dorn, 1954; Luthy et al., 1980; Ishikawa et al., 2002; Ginter and Mohamed, 2002), various aluminum alloys (Kim et al., 2000; Bae and Ghosh, 2002) and aluminum composites with SiC particulates (Spigarelli et al., 2002; Ma and Tjong, 2000), however experimental data for die-cast alloys are sparse. Since the die-casting process is extremely variable from manufacturer to manufacturer (in regards to gating methods, mold sizes, pressures and temperatures) and since die-cast alloys have been traditionally known to exhibit poor creep resistance, the study of die-cast aluminum alloys has been largely neglected by academia in favor of alloys that are theoretically more creep resistant, such as the nickel based superalloys (Nabarro and De Villiers, 1995). However, the need for the die-casting industry to acquire adequate analysis tools, and to obtain reliable data has heightened as current market-driven performance demands have escalated. Achievement of elevated design goals requires the development of robust constitutive models describing the die-casting industry specific materials. These expressions can then be used as inputs for finite element analysis and other methods used for design and development. Constitutive equations are powerful tools because of their general form, however application of these equations can be challenging when the description of complex behavior requires many parameters. This paper demonstrates a way with limited testing to determine the parameters which robustly predict inelastic behavior in the context of nonlinear viscoelastic superposition.

Creep and stress relaxation can both lead to degraded engine performance. Component failure or loss of tolerance, occurring largely in bolted joints, can contribute to engine inefficiency through seal leakage and piston blow-by. The casual mechanisms for both creep and relaxation involve diffusion, dislocation motion and grain boundary sliding, however, the details of these mechanisms are beyond the scope of this work. Herein, we concentrate on obtaining a robust constitutive description for the complex material behavior by employing a constitutive model based on single integral, nonlinear superposition to describe the creep and stress relaxation response. The method presented is not limited to the aluminum alloy studied here but is broadly applicable to other nonlinear materials, such as ligaments (Oza et al., 2001), and other materials, such as polymers, which exhibit primary creep. The method used here is not intended for large deformation plasticity.

1.2. Inelastic, Time Dependent Behavior

Metals at high homologous temperature (temperature on absolute scale divided by the melting temperature) greater than about 0.6 exhibit substantial time dependent behavior. At sufficiently small strain, typically below 10^{-5}, behavior follows linear viscoelasticity (Nowick and Berry, 1972), and is recoverable, therefore is referred to as anelastic. If the strain is moderately high, the behavior is nonlinear: the modulus as a measure of stiffness depends on strain and the time dependence depends on strain.

In linearly viscoelastic solids, one can readily interrelate creep and relaxation via Laplace transformations. Several interrelations have been presented for nonlinear time dependence but they generally do not involve superposition and they are inapplicable to primary creep. The interrelation of Ashby and Jones (1980), for example, assumes secondary creep. The rate of strain ε is assumed to be a power law in stress σ, $d\varepsilon/dt = B\sigma^n$. This does not allow for primary creep, which we observe in the present study. The total strain ε is regarded as the sum of an elastic part $\varepsilon_{\text{elastic}} = \sigma/E$ and a creep part $\varepsilon_{\text{creep}}$. Also, the approach does not reduce to linearly viscoelasticity at small strain. Specifically, the creep is nonlinear for all stress levels with no linear term. Ashby and Jones differentiate, substitute, then integrate over a range, to obtain the following relation for stress relaxation

$$\sigma_f(t) = \left\{ BE(n-1)t + \left(\frac{1}{\sigma_0^{n-1}} \right) \right\}^{-\frac{1}{n-1}}.$$

The resulting stress relaxation is not particularly realistic. It is nearly constant at short time and for $n = 2$ and for long time t, it goes as $1/t$. Other methods for secondary but not for primary creep were reviewed by Popov (1947) who also developed an implicit interrelation for a specific constitutive behavior separable into a product of time-dependent and stress dependent terms. However, Lakes and Vanderby (1999) showed by a superposition method that a separable form for creep leads to a non-separable form for relaxation. Rate-based studies make the simplifying assumption that the same relation between stress and creep strain rate is valid both under conditions of constant stress (creep) and constant strain (relaxation). Such an assumption is not likely to be valid for real materials. Touati and Cederbaum (1997) presented a complex and laborious numerical method to convert the separable creep model of Schapery (1969) into a set of first order nonlinear equations to predict relaxation. Again, Lakes and Vanderby (1999) showed that a separable form for creep leads to a non-separable form for relaxation.

Viscoplasticity models for metals also make use of strain rate in the constitutive equations. Many such models are reviewed by Lemaitre (2001). Emphasis is placed upon deformations of at least several percent strain under constant strain rate. For example, Bodner and Partom (1975) suggested a model in which the plastic strain rate is expressed as a nonlinear function of effective stress. Model parameters are extracted from monotonic stress strain curves at different strain rates (Chan et al., 1988). Refinements, presented by Bodner (2001) include temperature effects, of effects temperature variation, and stress reversals. Ellyin (2001) studied such rate effects in connection with numerical analysis of yield.

The viscoplasticity approach is appropriate for metals at large strain and relatively large strain rates from 10^{-3} to 10^4 sec^{-1}. A superposition approach is adopted here since it reduces properly to linear viscoelasticity at small strain, and since it allows a transparent interrelation between creep and stress relaxation. In the long term creep tests presented here, strain rates are 10^{-10} to 10^{-9} $\sec^{-1}$ so use of the rate control approach used in viscoplasticity would be problematical.

1.3. Nonlinear Superposition

The most general constitutive equations describing the time dependent stress and time dependent strain for linear viscoelastic materials are Boltzmann integrals, in which t is time, $J(t)$ is the time dependent creep compliance and $E(t)$ is the time dependent relaxation modulus. For linear materials there is no strain dependence on the relaxation modulus or stress dependence on the creep compliance as shown in Equation (1a) and (1b).

$$\sigma(t) = \int_0^t E(t-\tau)\frac{\mathrm{d}\varepsilon(\tau)}{\mathrm{d}\tau}\mathrm{d}\tau \tag{1a}$$

$$\varepsilon(t) = \int_0^t J(t-\tau)\frac{\mathrm{d}\sigma(\tau)}{\mathrm{d}\tau}\mathrm{d}\tau \tag{1b}$$

Some nonlinear materials may behave linearly, or at least appear to, for small strains and low temperatures especially if the time window is small. Material nonlinearities can manifest themselves even at moderate strains, but nonlinear effects take the forefront as the operating window is pushed to its extremes in stress/strain, temperature and operating life-time. If one is to consider a nonlinearly viscoelastic material the strain and stress dependence on the relaxation modulus and creep compliance respectively must be included in the time integral.

The nonlinear superposition method considered in this work, expressed as Equation (2) for stress relaxation and Equation (3) for creep, allows the relaxation modulus, E, and creep compliance, J, to be not only functions of time but also of applied strain and stress respectively.

$$\sigma(t) = \int_0^t E(t-\tau, \varepsilon(\tau))\frac{\mathrm{d}\varepsilon(\tau)}{\mathrm{d}\tau}\mathrm{d}\tau, \tag{2}$$

$$\varepsilon(t) = \int_0^t J(t-\tau, \sigma(\tau))\frac{\mathrm{d}\sigma(\tau)}{\mathrm{d}\tau}\mathrm{d}\tau. \tag{3}$$

These nonlinear relations are a special case of a general nonlinear expansion of a nonlinear functional (Green and Rivlin, 1957; Lockett, 1972; Findley et al., 1976), applicable to systems of any degree of nonlinearity. They are superposition integrals, capable of handling arbitrary stress or strain histories including creep, relaxation, recovery, multiple steps, and constant strain rate. A constant strain rate experiment can be modeled by letting the strain in Equation (2) exhibit a linear dependence on time.

This study makes use of the nonlinear superposition relations, Equations (2) and (3) to (i) interpolate between empirical creep curves and (ii) interpolates between two relaxation curves using an assumed constitutive equation and a 3-point isochronal method, and (iii) predict the relaxation response from an empirically fit creep curve using nonlinear superposition for the interrelationship. Validation of the use of nonlinear superposition is important technologically since mechanical testing can be expensive and time consuming. Thus, the ability to accurately determine the viscoelastic response for any input condition from a small number of empirical curves will decrease design costs and increase product quality and safety. Additionally, accurate prediction of the losses in a bolted joint, where both creep and relaxation occur, often requires intimate knowledge of the material's response between empirical curves.

2. Methods

2.1. Interpolation Between (i) Creep and (ii) Stress Relaxation Time Histories

A numerical procedure for both the interpolation techniques is included in the Appendix. A discussion of the major points and ideas is conducted here to elaborate on the rationale. Both the interpolation and interrelationship methods utilize isochronal curves in which points on the stress-strain curves are taken at constant time and temperature. Each point on an isochronal curve is obtained from a different creep or relaxation test.

2.1.1. *Creep Compliance*

The creep compliance of a nonlinear time dependent material may be written as a power series in time t and stress σ as follows.

$$J(t, \sigma) = g_1 + g_2\sigma^p t^n + g_3\sigma^q t^m + \cdots \tag{4}$$

The first term represents elastic behavior. Higher terms represent time dependence with the possibility of linear viscoelasticity or nonlinear viscoelasticity or inelastic behavior. Since the complexity of the interrelation grows rapidly with the order of the term (Oza et al., 2003), it is expedient to truncate the series to the minimum length consistent with the data at hand. For the present study, two terms suffice.

A 3-point isochronal method, described in Appendix A, is used to obtain g_1, g_2, p and n for Equation (4), from the experimental data. The value of 'p' that we have chosen is 0.75 as it was found to fit the isochronal data points for the present alloy well.

Since the strain is $\varepsilon = J(t, \sigma)\sigma$, we get (understanding the stress is constant in creep)

$$\varepsilon = g_1\sigma + g_2\sigma^{1.75}t^n \tag{5}$$

Data were collected for both creep and relaxation at different stress and strain levels respectively (Jaglinski and Lakes, 2004). Isochronals at three different times are fitted to the experimental data with the above equation via methods in the Appendix. The chosen form for creep compliance is non-separable since it cannot be factored into a product of stress dependence and time dependence.

2.1.2. *Relaxation Modulus*

The stress-strain values at different times are derived from relaxation data. Equation (6) is the assumed form for the relaxation modulus to be used for the interpolation method. Again, as for creep interpolation, the 3-point isochronal method is used to obtain f_1, f_2 and n from Equation (6).

$$E(t, \varepsilon) = f_1 \varepsilon^{0.05} + f_2 \varepsilon^{0.75} t^{-n} \tag{6}$$

Values of 0.05 and 0.75 are chosen for powers of strain as they were found to fit the isochronal data points for the present alloy well. Analysis of other materials via the present method would involve different exponents, to be extracted from the data. For the present experimental results, $n < 1$ expresses the fact the creep is primary creep. Details of the method and values obtained after fitting the relaxation data are given in Appendix B. This interpolation is not related to the interrelation of creep and relaxation which is developed in Section 2.2.

Since the stress is $\sigma = \varepsilon E(t, \varepsilon)$ it may be written

$$\sigma = f_1 \varepsilon^{1.05} + f_2 \varepsilon^{1.75} t^{-n} \tag{7}$$

2.2. Interrelation of Stress Relaxation from Creep Using Nonlinear Superposition

The single-integral constitutive equations used are Equation (2) for relaxation and Equation (3) for creep. Time-dependent strain due to constant creep stress can be written as a sum of immediate and delayed Heaviside step functions in time $H(t)$

$$\varepsilon(t) = \varepsilon(0)H(t) + \sum_{i=0}^{N} \Delta\varepsilon_i H(t - t_i) \tag{8}$$

Each step strain in the summation gives rise to a relaxing component of stress in view of the definition of the relaxation function. Decomposition of creep into step strains and generation of relaxation due to each step strain are shown in the top and bottom diagrams of Figure 1. Nonlinearity is accommodated in this analysis since the relaxation function E explicitly depends on strain level.

$$\sigma = \varepsilon(0)E(t, \varepsilon(0)) + \sum_{i=0}^{N} \Delta\varepsilon_i E(t - t_i, \varepsilon(t_i)) \tag{9}$$

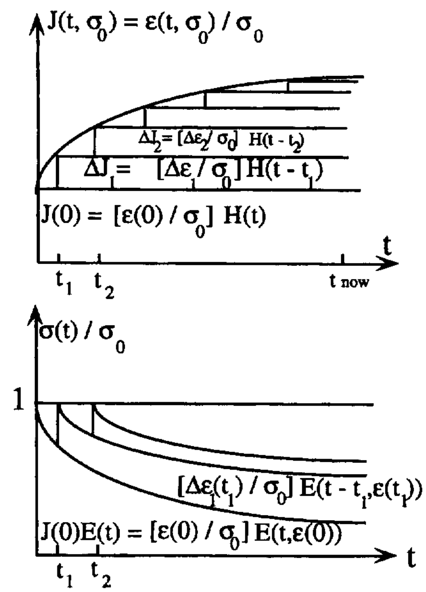

Figure 1. Top: decomposition of a creep function $J(t, \sigma_0)$ as a sum of immediate $H(t)$ and delayed Heaviside step function $H(t - t_i)$ in time t. Bottom: the constant stress 'σ_0' which is the same as 'σ' in the text gives rise to creep expressed as a sum of relaxing components, each of which comes from a step function in the decomposition of the creep curve above.

Here we assume there is no effect from interactions between the step components, hence we consider single-integral type nonlinear response (Lakes and Vanderby, 1999) and exclude responses which must be describable by a multiple integral formulation.

Dividing by σ and using the definition of creep compliance,

$$1 = J(0, \sigma)E(t, \varepsilon(0)) + \sum_{i=0}^{N} \Delta J_i E(t - t_i, \varepsilon(t_i)) \tag{10}$$

Pass to the limit of infinitely many fine step components to obtain a Stieltjes integral, with τ as a time variable of integration,

$$1 = J(0, \sigma)E(t, \varepsilon(0)) + \int_0^t E(t - \tau, \varepsilon(\tau)) \frac{\partial J(\tau, \sigma)}{\partial \tau} \mathrm{d}\tau. \tag{11}$$

The creep compliance J is a function of time and stress. As in the linear interrelation, time dependence appears in the integral as dependence on a time variable of integration. Since for creep under constant stress, $\sigma(t) = 0$ for $t < 0$ and $\sigma(t) = \sigma$

for $t > 0$, we have $\varepsilon(t) = \sigma J(t, \sigma)$, so Equation (11) becomes

$$1 = J(0, \sigma)E(t, \sigma J(0, \sigma)) + \int_0^t E(t - \tau, \sigma J(\tau, \sigma))\frac{\partial J(\tau, \sigma)}{\partial \tau}\mathrm{d}\tau. \tag{12}$$

To develop an explicit relationship between creep and relaxation, one assumes a particular functional form for one of the viscoelastic functions. For example, Lakes and Vanderby (1999) used this Stieltjes integral to show that a separable form of creep (e.g. $J(t, \sigma) = j(t)g(\sigma)$) gives rise to a non-separable relaxation function as described earlier. Other interrelations in the literature are discussed in Oza et al. (2003).

Use of this formulation below provides validation of the model and will allow the reduction of the required number of experiments needed to thoroughly map a material's response over a range of stress, temperature and time, thus minimizing experimental costs.

2.3. Two Term Formulation

We use an explicit form for creep compliance, similar to Equation (4) and using the semi-inverse method in the formulation below, we show that the corresponding assumed form of relaxation modulus predicts relaxation from creep. In this case, assumption of a purely elastic term is not possible in the creep compliance equation due to mathematical constraints in the formulation. So the power of time of the first term in Equation (13) is assumed to be very small. This makes the first creep compliance term, a quasi-elastic term.

Assume the creep behavior to be as follows, and restrict the power of stress in creep compliance to 0.75 throughout the analysis. The value of 0.75 was obtained from fitting of the nonlinear behavior of the metals in the present study. The value 0.001 was obtained from the observation that the stress-independent part of the compliance in these metals was virtually elastic (time independent) within the experimental resolution. A small nonzero value was used for the exponent since the interrelation equations are available for explicit power laws and since materials always exhibit some creep even at arbitrarily small stress.

$$J(t, \sigma) = g_1 t^{0.001} + g_2 \sigma^{0.75} t^m \tag{13}$$

Assume a non-separable power law form of relaxation, given as:

$$E(t, \varepsilon) \approx f_1 t^{-0.001} + f_2 \varepsilon(t)^{0.75} t^{-q} \tag{14}$$

in which f_1, f_2 and q are to be determined by the analysis.

The interrelation is based on the single Stieltjes integral form of Equation (11).

The creep function is differentiated and the functional forms substituted. Stress independent terms are used to develop the relation between f_1 and g_1, $\sigma^{0.75}$ terms are used to develop the relation between f_2 and g_2 and higher order stress terms (σ^n) are used to develop the relation between f_n and g_n. Since we have a two-term form of creep compliance and relaxation modulus, we ignore terms greater than $\sigma^{0.75}$.

While solving the Stieltjes integral, we equate all 'σ' independent terms to 1 to solve for f_1 and all the '$\sigma^{0.75}$' terms to 0 to solve for f_2.

Then,

$$E(t, \varepsilon(t)) = f_1 t^{-0.001} + f_2 g_1^{0.75} \sigma^{0.75} t^{-q+0.00075} \tag{15}$$

Since $J(0) = 0$, the first term in the Stieltjes integral vanishes.

Substituting (15) and the derivative of the creep compliance in the Stieltjes integral,

$$1 = \int_0^t \{f_1(t-\tau)^{-0.001} + f_2 g_1^{0.75} \sigma^{0.75} (t-\tau)^{-q+0.00075}\}\{0.001 g_1 \tau^{-0.999} + m g_2 \sigma^{0.75} \tau^{m-1}\} \mathrm{d}\tau \tag{16}$$

Equation (16) is of the form,

$$1 + 0 * \sigma^{0.75} = a + b\sigma^{0.75} \tag{17}$$

$$\text{So}\, a = 1 \quad \text{and} \quad b = 0. \tag{18}$$

Since Equation (16) is of the same form as Equation (17) and based on Equation (17) and its solution in Equation (18), we equate 'σ' independent terms in Equation (16) to 1 and all '$\sigma^{0.75}$' terms in Equation (16) to 0.

From Equation (16), we get

$$1 = f_1 g_1 \int_0^t 0.001(t-\tau)^{-0.001} \tau^{-0.999} \mathrm{d}\tau.$$

$$1 = f_1 g_1 \frac{1}{\sin 0.001\pi} 0.001\pi \tag{19}$$

where $\frac{1}{\sin 0.001\pi} 0.001\pi \approx 1$.

So $f_1 \cong \frac{1}{g_1}$ as in the linear case.

Now we take all the '$\sigma^{0.75}$' terms

$$0 = f_1 g_2 \int_0^t m(t-\tau)^{-0.001} \tau^{m-1} \mathrm{d}\tau + f_2 {g_1}^{1.75} \int_0^t 0.001(t-\tau)^{-q+0.00075} \tau^{-0.999} \mathrm{d}\tau$$

$$0 = f_1 g_2 m \left\{ \frac{\Gamma(0.999)(m)}{\Gamma(m+0.999)} \right\} t^{m-0.001} + f_2 {g_1}^{1.75} 0.001 \left\{ \frac{\Gamma(-q+1.00075)\Gamma(0.001)}{\Gamma(-q+1.00175)} \right\} t^{-q+0.00175} \quad (20)$$

Here Γ is the gamma function. To account for time-dependence, powers of the time terms must be the same.

$$\begin{aligned} m - 0.001 &= -q + 0.00175 \\ q &= 0.00275 - m \end{aligned} \quad (21)$$

Substituting (21) in (20), we obtain

$$0 = f_1 g_2 m \left\{ \frac{\Gamma(0.999)(m)}{\Gamma(m+0.999)} \right\} t^{m-0.001} + f_2 {g_1}^{1.75} 0.001 \left\{ \frac{\Gamma(0.998+m)\Gamma(0.001)}{\Gamma(m+0.999)} \right\} t^{m-0.001}$$

Canceling the common terms from the above equation and solving for f_2, we obtain

$$f_2 = \frac{-f_1 g_2 m \Gamma(0.999)\Gamma(m)}{0.001 {g_1}^{1.75} \Gamma(0.998+m)\Gamma(0.001)} \quad (22)$$

Since $\Gamma(0.001) \approx 1000$ and $\Gamma(0.999) \approx 1$, Equation (22) becomes

$$f_2 \cong \frac{-f_1 g_2 m \Gamma(m)}{{g_1}^{1.75} \Gamma(0.998+m)} \quad (23)$$

Values of f_1 and f_2 obtained can be used in Equation (14) to predict the relaxation curve.

2.3.1. *Experimental Methods*

All test specimens were supplied by commercial die-cast component manufacturers. As-cast tensile specimens were used with gauge dimensions of 2.5″ long × 0.25″ diameter and 2.5″ long × 0.245″ diameter due to two different specimen sources. The alloy compositions in weight percent were as follows: B390 as Al–17Si–4Cu–0.5Mg, Eutectic Al–13Si–3Cu–0.2Mg, (hereafter referred to as eutectic) and Al–17Si–0.2Cu–0.5Mg–1.2Fe (referred to hereafter as Al–17Si).

Constant strain stress relaxation tests were conducted on an MTS servo-hydraulic test frame (20,000 lb MTS, Minneapolis, MN). A Lindberg furnace with integral controller provided temperature control. Micromeasurements WK-13-250BG-350Ω strain gauges were used for creep and relaxation testing. Strain gauge conditioning was handled by the internal signal conditioner, however due to the small bridge balancing range of the internal conditioner an extra in-line (between the sample and the conditioner) bridge balancing resistor was added to handle coarse adjustments. Excitation was set by using the shunt calibration method at full scale, or $1\ \mathrm{V} = 1 \times 10^{-3}$ strain for a total of 1×10^{-2} strain for 10 V.

All relaxation tests were conducted in strain control mode set at the 20% range or $1\ \mathrm{V} = 2 \times 10^{-4}$ strain. For sample installation and test warm-up, however, the machine was set to stroke control to maintain actuator position due to thermal drift in the bridge circuit while bringing the sample to temperature. The sample was brought to temperature in the testing fixture and maintained at steady state for at least 12 hr prior to application of strain.

The initial 200 sec were captured using a Tektronix TDS420A oscilloscope. Data after 200 sec were taken manually by reading the digital output on the MTS frame in such a way as to approximate logarithmic time steps. Data are not reported until after the first 10 sec of each test since the rise-time of the load history was about 2 sec. Relaxation tests were run for 7 days. Strain levels chosen for relaxation corresponded to the stresses run for creep.

All the three aluminum–silicon alloys i.e. B390, eutectic Al–Si alloy and Al–17Si were tested at 220 °C at 31, 57 and 73 MPa for creep and 430×10^{-6}, 850×10^{-6} and 1200×10^{-6} strain for relaxation. The rise time was 2 sec for creep. Stress or strain versus time data were plotted on a log–log scale. Isochronals (stress-strain curves at a particular time) were used for curve fitting as described in Appendices A and B.

3. Applications of the Nonlinear Superposition

3.1. Interpolation Between Empirical Creep-Strain and Stress-Relaxation Time Histories

Results from the eutectic Al–Si alloy are presented in Figure 2 for creep and Figure 3 for relaxation. Equation (4) is used to model the creep compliance. Isochronals are created from the creep tests done at 31 and 73 MPa. g_1, g_2 and n are obtained by fitting three different isochronals as discussed in Appendix A. These values are then used to predict a creep curve at an intermediate level of 57 MPa. A correlation (R) of 0.96 was obtained (via KaleidaGraph software (Synergy Software, 2457 Perkiomen Avenue, Reading, PA 19606, USA)) for a prediction of a creep curve done at 57 MPa. Correlation (R) is a measure of the strength of association between two variables (Kocher and Zurakowski, 2004).

For the relaxation modulus modeling, Equation (6) is used. Isochronals are created from the relaxation tests done at 430×10^{-6} and 1200×10^{-6} strain. f_1, f_2 and n are obtained by fitting three different isochronals as discussed in Appendix B.

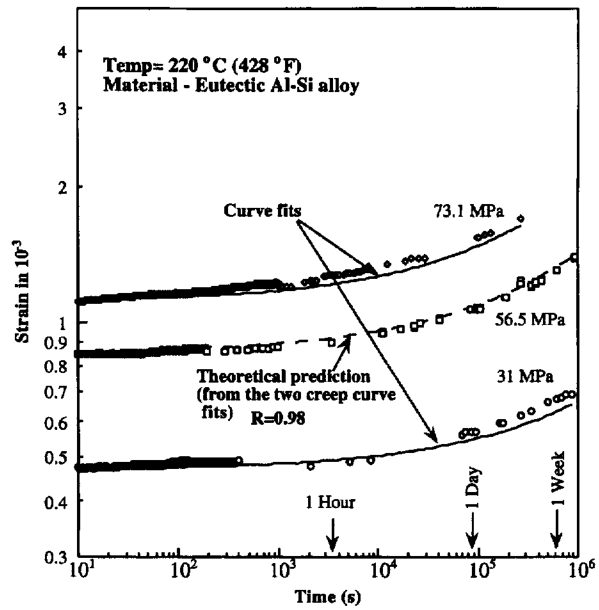

Figure 2. Three creep curves (31, 56 and 73 MPa) at a constant temperature of 220 °C are plotted against time. Creep at 31 and 73 MPa are fitted with $\varepsilon = g_1\sigma + g_2\sigma^{1.75}t^n$ where the coefficients are obtained from isochronals of the stress levels. Using the obtained coefficients, creep at 56 MPa is predicted very well.

These values are then used to predict a relaxation curve at an intermediate level of 850×10^{-6} strain. A correlation (R) of 0.98 was obtained for a prediction of a relaxation curve done at 850×10^{-6}.

So, in this technique, creep tests at two different stress levels (31 and 73 MPa) are curve-fitted using the method given in Appendix A to obtain g_1, g_2 and n. These coefficients are substituted in Equation (4) to predict the creep response at a stress level of 57 MPa.

Similarly, relaxation tests at two different strain levels (430×10^{-6} and 1200×10^{-6}) are curve-fitted using the method given in Appendix B to obtain f_1, f_2 and n. These coefficients are substituted in Equation (6) to predict the relaxation response at a strain level of 850×10^{-6}.

It should be noted that the values g_1, g_2, f_1, f_2, and n are used only for the interpolation scheme and a new set of values are determined for the interrelationship.

3.2. Interrelation of Stress Relaxation from Creep

Creep curves at 31 and 57 MPa are fitted with Equation (4) for creep compliance using the method explained in Appendix A. Figure 4 shows the theoretical curve fit obtained for creep curves at stress levels of 31 and 57 MPa.

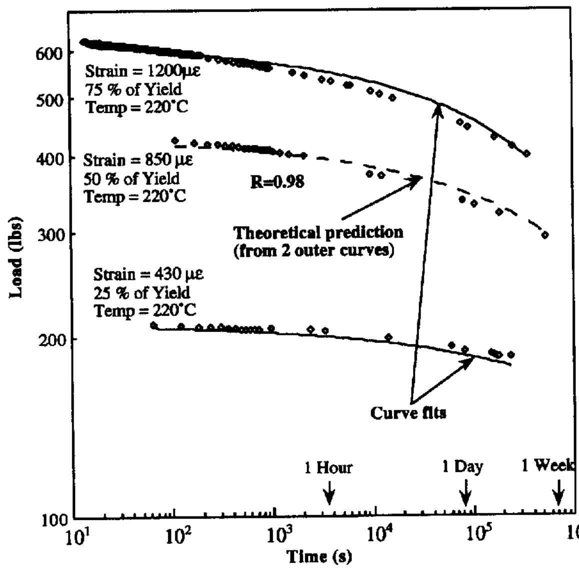

Figure 3. Three relaxation curves (430×10^{-6}, 850×10^{-6} and 1200×10^{-6} strain) at a constant temperature of 220 °C are plotted against time. Relaxation curves at 430×10^{-6} and 1200×10^{-6} are fitted with $\sigma = f_1\varepsilon^{1.05} + f_2\varepsilon^{1.75}t^{-n}$ where the coefficients are obtained from isochronals of the strain levels. Using the same coefficients, relaxation at 850×10^{-6} is predicted very well.

Using the interrelation formulation given in Section 2.2, coefficients for corresponding relaxation modulus (Equation 6) were determined. Using Equation 6 for the relaxation modulus and coefficients calculated by the interrelation, relaxation was then predicted for strains at 430×10^{-6} and 850×10^{-6}. Due to the nature of interrelated parameters, the shape of the creep curve and the shape of the relaxation curve are explicitly dependent on each other. Thus a small change in the input parameters used in the equation to fit the creep curve, produce a large change in the interrelated parameters of the relaxation modulus.

4. Results and Discussion

The goal of this study was to interpolate between empirical creep-strain and stress-relaxation time histories and to determine the stress relaxation response from corresponding creep data. Herein we have developed analytical methods based on nonlinear superposition to robustly describe creep and relaxation response. It was shown that any creep or relaxation at an intermediate stress or strain level can be predicted by using the coefficients obtained from the 3-point isochronal method.

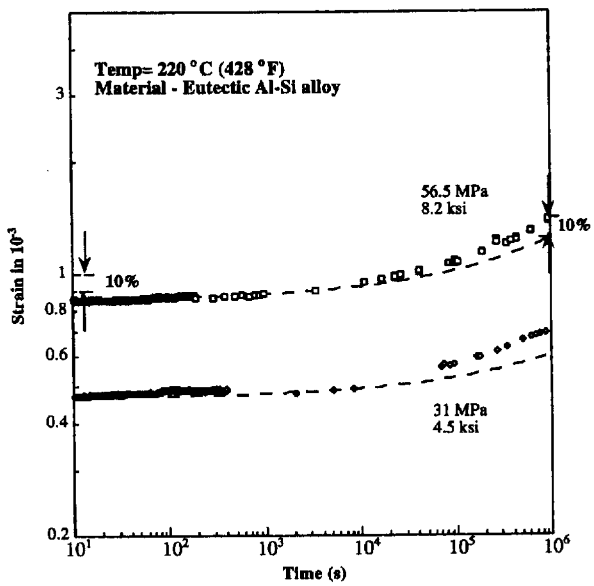

Figure 4. Two creep curves (31 and 56 MPa) at a constant temperature at 220 °C are plotted against time. Creep at 31 and 56 MPa are fitted with $\varepsilon = g_1\sigma + g_2\sigma^{1.75}t^n$ where the coefficients are obtained from isochronals of the stress levels.

Very high correlations of 0.98 and 0.98 are obtained using nonlinear superposition shown in Figures 2 and 3 for creep and relaxation respectively. The forms used in Equations (5) and (7) to fit and interpolate between creep and relaxation curves respectively are very simple and give good results for a wide range of stress levels for creep and strain levels for relaxation. The interpolation technique was also used to fit creep data at 92 MPa. Since the model we have used has only one stress-dependent term, there is a window restriction of stress and time. Moreover, stress levels of 92 MPa are quite unrealistic during normal engine operating conditions since 92 MPa is approximately 80% of the yield stress at 220 °C. So, the method we have used works very well over a practical envelope of realistic stress and strain levels.

In this work, creep and stress relaxation are also interrelated for primary creep described by a sum of power-law terms in time, within the framework of single integral nonlinear superposition. Figure 5 shows relaxation curves predicted from creep using the formulation above. Predicted results lie within 10% of experimental data. However, one reason for the offset of the prediction from the experimental data lies in the fact that the applied strains for the relaxation tests did not exactly correspond to the applied stresses for the equivalent creep tests. Also, the model for creep and relaxation that we have used for comparison with experiment is first

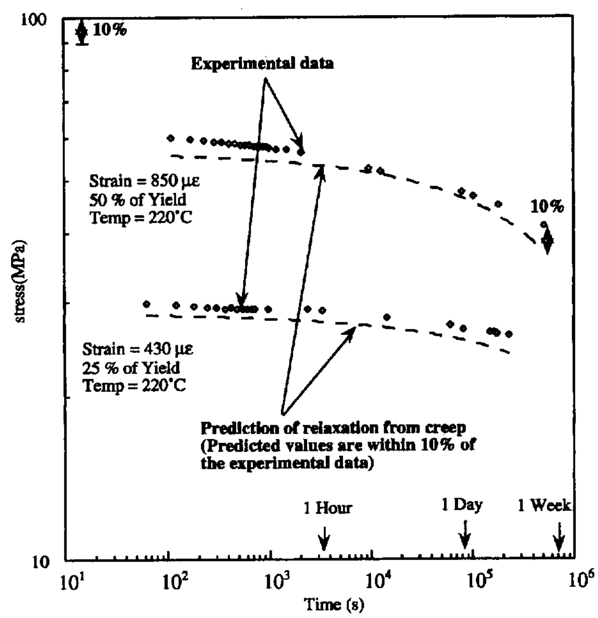

Figure 5. Prediction of relaxation from creep; comparison with experimental relaxation of alloy. The two corresponding strain levels for relaxation are ε_1 (430×10^{-6}) and ε_2 (850×10^{-6}). Similarly relaxation data of aluminum for two different strain levels are predicted very well by the interrelation used in this formulation. The points in the figure give the experimental curve, while the solid line is the theoretical prediction.

order in stress dependence. We have fitted the creep curve for six decades of time in seconds as shown in Figure 4 and predicted relaxation for six decades of time in seconds as shown in Figure 5. Even linear viscoelastic materials display a creep response with longer time constant than that for relaxation. One anticipates that a longer creep test must be used to predict relaxation for a shorter time for weakly nonlinear time dependent solids as well.

Simplicity in the phenomenological viscoelastic model and analytical interrelationship developed herein can be used to model more robustly many complex viscoelastic materials in the regime of weakly nonlinear time-dependence, and to reduce the number of tests required to characterize both creep and relaxation.

5. Conclusion

Experimental results for two Al–Si alloys disclosed primary creep behavior consistent with nonlinear superposition. Results for intermediate values of stress and strain were well modeled by a constitutive formulation based on upper and lower values.

Relaxation predicted from creep was within 10% of observed relaxation.

Appendix A

To construct the isochronals for purposes of interpolation and interrelation, two empirical creep curves at different levels of applied stress, but at constant material composition and temperature, are required.

The equation chosen for creep is

$$\varepsilon = g_1\sigma + g_2\sigma^{1.75}t^n \tag{A1}$$

We will construct three isochronals, which for example can be at time $t = 0$, 1 and any other point in time.

Three steps are required to find the values of g_1, g_2 and n. [1] The isochronal for $t = 0$ is curve fitted with $\varepsilon = g_1\sigma$ from Equation (A1) to get g_1, [2] the isochronal for $t = 1$ is curve fitted with $\varepsilon = g_1\sigma + g_2\sigma^{1.75}$ from Equation (A1) and the value of g_1 obtained from step [1] to get g_2, and [3] the isochronal for $t = x$ where x is some point in time and the value of g_1 and g_2 are substituted into Equation (A1) to obtain n.

Since there is no data point at $t = 0$ or 1 sec due to the experimental rise time, the first and second point in time for the isochronals were selected at 10 and 5000 sec respectively. Based on Figure 2, three points (low strain, mid strain and high strain) had to be selected for the isochronals so that the entire creep regime is covered. The first and second point in time for the isochronals at 10 and 5000 sec represent points in the low and mid strain region respectively.

We substitute $t = (t_n - 10)/4990$ into Equation (A1), where t_n are the actual times. So at $t_1 = 10$ sec and using the relation $t = (t_n - 10)/4990$ we get $t = 10 - 10/4990$ sec which is equal to 0. So $t = 0$ at $t_1 = 10$ sec. Similarly, we obtain $t = 1$ at $t_2 = 5000$ sec. For our case, the last point in time for the isochronal that we selected is $t_x = 90000$ sec and using the same above relation we get $t = 18.03$ sec.

Appendix B

The procedure for isochronal construction and curve fitting to describe relaxation is basically the same as that used in Appendix A. This procedure is applied to curve fit relaxation curves in Figure 3.

However, the equation used for relaxation curves is

$$\sigma = f_1\varepsilon^{1.05} + f_2\varepsilon^{1.75}t^{-n} \tag{A2}$$

Again, three isochronals are constructed at $t = 0$, 1 and any other point in time (x). Similarly, the three step process is as follows: [1] The isochronal for $t = 0$ is fitted with $\sigma = f_1\varepsilon^{1.05}$ from Equation (A2) to obtain f_1, [2] the isochronal for $t = 1$ is fitted with $\sigma = f_1\varepsilon^{1.05} + f_2\varepsilon^{1.75}$ and the value of f_1 to determine f_2, and [3] the isochronal at $t = x$ and known values of f_1 and f_2 are substituted into Equation (A2) to obtain n.

Again, no data exists at $t = 0$ or 1 sec, so the first and second point in time for the isochronals selected were at 14 and 190 sec respectively. Based on Figure 3, the three points (low stress, mid stress and high stress) had to be selected for the isochronals so that the entire relaxation regime is covered. The first and second point in time for the isochronals at 14 and 194 sec represent points in the high and mid stress region respectively. Any other points in time also could be selected.

So we substitute $t = (t_n - 14)/176$ into Equation (A2) where t_n are the actual times. This gives $t = 0$ at $t_1 = 14$ sec and $t = 1$ at $t_1 = 190$ sec.

So at $t_1 = 14$ sec and using the relation $t = (t_n - 14)/176$ we get $t = 14 - 14/176$ which is equal to 0.

So $t = 0$ at $t_1 = 14$ sec.

Similarly, we obtain $t = 1$ at $t_2 = 190$ sec.

For our case, the last point in time for the isochronal that we selected is $t_x = 2.5 \times 10^5$ sec and using the same above relation we get $t = 1420.3$ sec.

References

Ashby, M.F. and Jones, D.R.H., *Engineering Materials*, Pergamon, Oxford, 1980.

Bae, D.H. and Ghosh, A.K., 'Cavity formation and early growth in a superplastic Al-Mg alloy', *Acta Mater.* **50**, 2002, 511–523.

Bodner, S.R. and Partom, Y., 'Constitutive equations for elastic-viscoplastic strain hardening materials', *ASME J. Appl. Mech.* **42**, 1975, 385–389.

Bodner, S.R., 'An outline of the Bodner-Partom (B-P) unified constitutive equations for elastic-viscoplastic behavior' in *Handbook of Materials Behavior Models, Vol 1 Deformations of Materials*, J. Lemaitre (ed.), Academic Press, San Diego, 2001, 349–357.

Chan, K.S., Bodner, S.R. and Lindholm, U.S., 'Phenomenological modeling of hardening and thermal recovery in metals', *ASME J. Eng. Mater. Technol.* **110**, 1988, 1–8.

Dorn, J.E., 'Some fundamentals on high temperature creep', *J. Mech. Phys. Solids* **3**, 1954, 85–116.

Ellyin, F., 'Rate-dependent elastoplastic constitutive relations', in *Handbook of Materials Behavior Models, Vol. 1 Deformations of Materials*, J. Lemaitre (ed.), Academic Press, San Diego, 2001, 377–386.

Findley, W.N., Lai, J.S. and Onaran, K., *Creep and Relaxation of Nonlinear Viscoelastic Materials*, North Holland, 1976.

Ginter, T.J. and Mohamed, F.A., 'Evidence for dynamic recrystallization during Harper-Dorn creep', *Mater. Sci. Eng.* **A322**, 2002, 148–152.

Green, E. and Rivlin, R.S., 'The mechanics of nonlinear materials with memory', *Arch. Ration. Mech. Anal.* **1**, 1957, 1–21.

Ishikawa, I., Maehara, M. and Kobayasi, Y., 'Mechanical modeling and microstructural observation of pure aluminum crept under constant stress', *Mater. Sci. Eng.* **A322**, 2002, 153–158.

Jaglinski, T. and Lakes, R.S., 'Creep behavior of Al–Si die-cast alloys', *J. Eng. Mater. Technol.* **26**, 2004, 378–382.

Kim, W.J., Yeon, J.H. and Lee, J.C., 'Superplastic deformatrion of spray-deposited hyper-eutectic Al–25Si alloy', *J. Alloys Compd.* **308**, 2000, 237–243.

Kocher, M.S. and Zurakowski, D., 'Clinical epidemiology and biostatistics: A primer for orthopaedic surgeons', *J. Bone Joint Surg.* **86-A**(3), 2004, 607–620.

Lakes, R.S. and Vanderby, R., 'Interrelation of creep and relaxation: A modeling approach for ligaments', *ASME J. Biomech. Eng.* **121**, 1999, 612–615.

Lemaitre, L. (ed.), *Handbook of Materials Behavior Models, Vol. 1 Deformations of Materials*, Academic Press, San Diego, 2001.

Lockett, F.J., *Nonlinear Viscoelastic Solids*, Academic Press, NY, 1972.

Luthy, H., Miller, A.K. and Sherby, O.D., 'The stress and temperature of steady-state flow of intermediate temperatures for pure polycrystalline aluminum', *Acta Metall.* **28**, 1980, 169–178.

Ma, Z.Y. and Tjong, S.C., 'High-temperature creep behavior of SiC particulate reinforced Al–Fe–V–Si alloy composite', *Mater. Sci. Eng.* **A278**, 2000, 5–15.

Nabarro, F.R.N. and de Villiers, H.L., *The Physics of Creep*, Taylor and Francis, London, 1995.

Nowick, A.S. and Berry, B.S., *Anelastic Relaxation in Crystalline Solids*, Academic Press, NY, 1972.

Oza, A.L., Vanderby, R. and Lakes, R.S., 'Interrelation of creep and relaxation in nonlinearly viscoelastic material: Application to ligament and metal', *Rheol. Acta* **42**, 2003, 557–568.

Popov, E.P., 'Correlation of tension creep tests with relaxation tests', *J. Appl. Mech.* **14**, 1947, 135–142.

Schapery, R.A., 'On the characterization of nonlinear viscoelastic materials', *Polym. Eng. Sci.* **9**, 1969, 295–310.

Servi, I.S. and Grant, N.J., 'Creep and stress rupture behavior of aluminum as a function of purity', *Trans. AIME* **191**, 1951, 909–916.

Spigarelli, S., Cabibbo, M., Evangelista, E. and Langdon, T.G., 'Creep properties of an Al-2024 composite reinforced with SiC particulates', *Mater. Sci. Eng.* **A328**, 2002, 39–47.

Touati, D. and Cederbaum, G., 'On the prediction of stress relaxation from known creep of nonlinear materials', *J. Eng. Mater. Technol.* **119**, 1997, 121–124.