Experimental Limits on the Photon Mass and Cosmic Magnetic Vector Potential

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A novel experimental approach based on a toroid Cavendish balance is used to evaluate the product of photon mass squared and the ambient cosmic magnetic vector potential A. The method is based on the energy density of the vector potential in the presence of photon mass, not on measurement of the magnetic field. The experiment discloses $A\mu_\gamma^2 < 2 \times 10^{-9}$ T m/m², with μ_γ^{-1} as the characteristic length associated with photon mass. Consequently, if the ambient magnetic vector potential is $A \approx 10^{12}$ T m due to cluster level fields, $\mu_\gamma^{-1} > 2 \times 10^{10}$ m. If we conservatively use galactic fields prior to a reversal, then $\mu_\gamma^{-1} > 1 \times 10^9$ m, a figure still superior to that derived from the Jovian magnetic field.
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The photon mass is ordinarily assumed to be exactly zero. If there is any deviation from zero, it must be very small, since Maxwellian electromagnetism has been very well verified (in the classical domain). A nonzero photon mass would give rise to a wavelength dependence of the speed of light in free space, the possibility of longitudinal electromagnetic waves, a leakage of static electric signals into conductive enclosures, and a more rapid (exponential or Yukawa) falloff of magnetic dipole fields with distance [1,2] than the usual inverse cube dependence.

Electromagnetism in the presence of nonzero photon mass is described by the Maxwell-Proca equations [1,3],

$$\text{div}\, \mathbf{E} = 4\pi\rho - \mu_\gamma^2 V, \tag{1a}$$

$$\text{curl}\, \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \tag{1b}$$

$$\text{div}\, \mathbf{B} = 0, \tag{1c}$$

$$\text{curl}\, \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} - \mu_\gamma^2 \mathbf{A}, \tag{1d}$$

in which $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic field, ρ is the charge density, $\mathbf{J}$ is the current density, V is the scalar potential, $\mathbf{A}$ is the vector potential, c is the speed of light, and $\mu_\gamma^{-1} = \hbar/m_\gamma c$ is a characteristic length, the Compton wavelength of the photon, with m_γ as the photon mass. Maxwell's equations correspond to $m_\gamma = 0$. The possibility of nonzero photon mass has been studied by Bass and Schrödinger [4], deBroglie and Vigier [5], Feynman [2], and others. Gauge invariance is lost [2] if $\mu_\gamma > 0$, since, in the Maxwell-Proca equations, the potentials themselves have physical significance, not just through their derivatives; the Lorentz gauge is required.

Several experimental limits on the photon mass have been reported. Laboratory measurements of the speed of light at different frequencies [2] give $\mu_\gamma^{-1} > 1.4$ km ($m_\gamma < 10^{-10}$ eV or 2×10^{-43} g), laboratory tests [2,6] of Coulomb's law give $\mu_\gamma^{-1} > 3.1 \times 10^7$ m, measurements [7] of Earth's magnetic field give $\mu_\gamma^{-1} > 1 \times$ 10^8 m, more recently [8] $\mu_\gamma^{-1} > 2.5 \times 10^8$ m, and measurements [9] of Jupiter's magnetic field give $\mu_\gamma^{-1} > 5 \times 10^8$ m (corresponding to a photon mass $m_\gamma < 6 \times 10^{-16}$ eV or 8×10^{-49} g). More stringent limits based on inference from large-scale magnetic features in astronomical plasma objects have been reported as reviewed by Barrow and Burman [1], but such inferences are very indirect in view of the uncertainty regarding the mechanism of generation of such fields. Photon mass has been suggested by Georgi, Ginsparg, and Glashow [10] to affect cosmic background radiation. Photon mass, a very low energy phenomenon, would constrain the structure of theories at arbitrarily high energies. Possible anisotropy of the speed of light with respect to the cosmic background radiation [11] may be linked to photon mass. The Heisenberg uncertainty principle gives the smallest measurable mass in a universe of finite age [1] (considered to be about 5×10^{17} sec), corresponding to $\mu_\gamma^{-1} \geq 1.5 \times 10^{26}$ m. Experimental study of photon mass is difficult since the length scale to be studied is so large: Either the experiment must interrogate a region of size comparable to μ_γ^{-1} or it must have extraordinary precision [2] [the fractional change in the field in a region of size D is of the order $(\mu_\gamma D)^2$] as in the concentric sphere experiment to test Coulomb's law [6]. The experiment reported here interrogates vector *potentials* (not fields) arising from large-scale weak magnetic fields on a galactic scale or larger. The present method, in contrast with purely astrophysical inferences, requires no assumptions about how the large-scale fields are generated.

The potentials V and $\mathbf{A}$, defined below in terms of the fields $\mathbf{E}$ and $\mathbf{B}$, are considered to be nonobservable in Maxwellian electromagnetism, since the energy density associated with them is zero,

$$\mathbf{E} = -\nabla V - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}, \tag{2a}$$

$$\mathbf{B} = \text{curl}\, \mathbf{A}. \tag{2b}$$

The potential **A** is observable in the Aharonov-Bohm effect [12], but only via its line integral, not pointwise. If the photon mass is nonzero, the potentials acquire a small density of energy [2] which admits the possibility of a pointwise measurement.

The present experiment incorporates a novel approach in which influence of a large cosmic magnetic vector potential **A** in the presence of photon mass is sought in the laboratory via the associated energy density [2] $\mu_\gamma^2 A^2$. A modified Cavendish balance (Fig. 1) was used to determine the product $A\mu_\gamma^2$. A toroid of electrical steel, of mass 8.4 kg, was wound with 1260 turns of wire carrying 37 mA of current and supported by water flotation [13]. Stability and a restoring torque were provided by a tungsten wire of diameter 0.23 mm and length 198 mm, annealed under tension to reduce drift [14]. Calculated wire structural rigidity is 2.1×10^{-4} N m/rad. Torque sensitivity of the suspension was tested using a permanent magnet (calibrated with a known aluminum alloy) fixed to the wire, in the field of a Helmholtz coil [15]. The observed sensitivity was equal to the calculated value, within error limits, and angular deformation was linear with coil current. A thinner wire was not used since resolution was limited by environmental noise. A fine copper wire provided the electrical return path. The device was placed in an enclosure to eliminate the effect of air currents. A magnetic shield of mu metal was added in an attempt to eliminate noise due to magnetic field fluctuation, which would have an effect on a nonideal toroid. The experiment was done with and without shielding. Noise of magnetic origin was not a problem. The shielded apparatus was insensitive to an external 0.5 G field. Rotational motion of the toroid was measured by an optical lever system in which a low power laser beam was reflected from a mirror upon the toroid to a position-sensitive silicon sensor (UDT Corp.). Angle sensitivity was 2.6 mrad/V, and low frequency environmental mechanical noise giving rise to 50 mV at the angle sensor limited the resolution. Sensor output was amplified and recorded by a digital data acquisition system. Data were collected in segments as long as one month over a period of 18 months. Electric current I in the toroid windings generates no external magnetic field, but it does generate a dipole field of vector potential **A**. That dipole field of **A** interacts with the ambient vector potential $\mathbf{A}_{\text{ambient}}$ to produce a torque τ on the toroid. To understand the method, observe that a current loop of radius r carrying current I gives rise to a magnetic dipole moment $\mathbf{m}_d = \pi r^2 I/c$ of **B** field via Eq. (1). Such a loop, immersed in a magnetic field **B**, experiences a torque $\tau = \mathbf{m}_d \times \mathbf{B}$ by virtue of the energy density $\sim \mathbf{B}^2$. In the present experiment, a toroid coil contains a loop of magnetic flux Φ, which acts as a dipole source $\mathbf{a}_d$ of vector potential **A** via Eq. (2b) with magnetic field within the toroid as the source term, formally identical to Eq. (1d) with current density **J** as the source term;
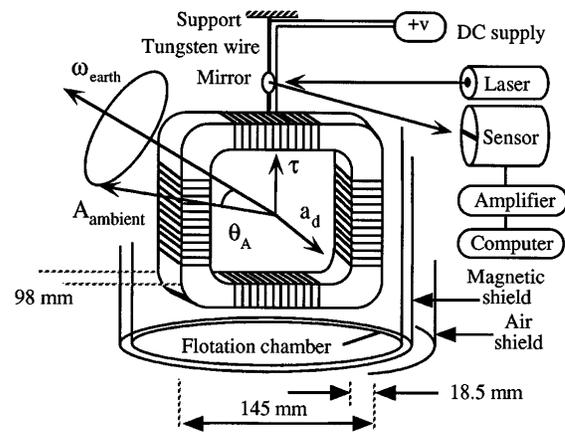

FIG. 1. A toroid carries an electric current giving rise to a dipole field $\mathbf{a}_d$ in magnetic vector potential. If $\mu_\gamma > 0$, this interacts with the ambient vector potential $\mathbf{A}_{\text{ambient}}$ to produce a torque τ on the toroid, which varies with time according to the rotation ω of Earth.

$\mathbf{a}_d = \pi r^2 \Phi/\mu_0$. The torque upon the **A** dipole is $\tau = \mathbf{a}_d \times \mathbf{A}_{\text{ambient}} \mu_\gamma^2$ via the energy density [2] of the vector potential. The torque gives rise to angular displacement ϕ_w of the wire, which has shear modulus G, diameter d, and length L; $\tau = G(1/L)(\pi d^4/32)\phi_w$. The observed angular displacement of the reflected laser beam is $\phi = 2\phi_w$. Setting the torque expressions equal and evaluating the magnetic flux in the toroid via its geometry and permeability $k = 5300$,

$$\mu_\gamma^2 |\mathbf{A}_{\text{ambient}}| = \frac{G \dfrac{1}{L} \dfrac{\pi d^4}{32} \dfrac{\phi}{2}}{k\left[\dfrac{1}{4}(w-u)^2 nhI \ln\left(\dfrac{w}{w-u}\right)\right]} \times \sin(\theta_A), \qquad (3)$$

in which θ_A is on the angle between $\mathbf{A}_{\text{ambient}}$ and Earth's rotation axis. The torque vanishes if the photon mass is zero or if the cosmic ambient vector potential were to be fortuitously aligned with Earth's rotation axis. Magnetic vector potentials from sources outside the solar system, within a laboratory frame of reference, appear to vary sinusoidally with time, one cycle per sidereal day, as a result of the rotation of Earth. Therefore, any signal due to them has a distinctive signature. Data were analyzed by fitting to the data a sinusoid of the required frequency, but unknown amplitude and phase, summed with a linear function to allow for drift due to slow evaporation of water. Disturbances due to human activity are synchronized to the solar day length rather than the sidereal day. Owing to the long-range nature of the potentials, this Cavendish balance acts as a cosmological compass, sensitive to the magnetic vector potential, in the presence of photon mass.

The largest source of uncertainty is the fact that neither the magnitude nor the direction of $\mathbf{A}_{\text{ambient}}$ can be specified with any precision. As for direction, the fraction

of solid angle which may be occupied by $\mathbf{A}_{\text{ambient}}$, which would give $\sin(\theta_A) \leq 0.1$, is 5×10^{-3}; therefore, an alignment which would desensitize the experiment significantly is highly fortuitous. As for magnitude, the magnetic field used to estimate $\mathbf{A}_{\text{ambient}}$ is not precisely known, and a cosmological map of magnetic field is not available. Nevertheless, we invoke a neo-Copernican argument to suggest that cancellation of vector potential to near zero (e.g., more than a factor of 10 from our estimate) in our immediate neighborhood would be highly fortuitous, since there is no reason that our immediate neighborhood should be anything but representative of the universe in the large, with the caveat that we are within a galaxy as described below. As for other possible errors, balances supported by water can "lock-up" due to resistance from adsorbed impurity layers upon the water, giving rise to a spurious zero signal. Lock-up was not a problem in our balance, since mechanical noise from the laboratory was detected in the angle signal. Any imperfection in the toroid which causes leakage of the magnetic field will give a static torque due to interaction with Earth's field rather than a torque which varies with time and so cannot mimic photon mass. The magnetic shielding cannot shield the desired $\mathbf{A}$ signal since the potentials couple very weakly with matter [2] (not at all if photon mass is zero) so the "permeability" of any shield to $\mathbf{A}$ is indistinguishable from unity. Error associated with uncertainty in the geometry of the toroid (3%) and wire is dominated by variation in the wire diameter as a result of annealing under tension. The wire's rigidity goes as d^4, so diameter variation causes a 9% error. Such error is negligible compared with the uncertainty in the potential.

Approximate contributions to the ambient vector potential $|\mathbf{A}_{\text{ambient}}| \equiv A$ are, following Eq. (2b), 200 T m ($1\,\text{T} = 10^4\,\text{G}$) due to Earth's magnetic dipole field, and 10 T m due to the Sun's magnetic dipole field. Galactic magnetic fields [16,17] are on the order of 1 μG and experience a reversal about 600 pc ($1\,\text{pc} = 3.08 \times 10^{16}\,\text{m}$) toward the center of the Milky Way; such a region gives $A \approx 2 \times 10^9$ T m. On yet larger distance scales, magnetic fields of 0.2 μG over a distance of 1300 kpc corresponding to the Coma galactic cluster [18] correspond to $A \approx 10^{12}$ T m. Magnetic fields of ≈ 1 μG over Mpc dimensions in galactic clusters are widespread [19]. Fields of 0.3 to 0.6 μG occur over 40 Mpc in a filament bridging two clusters [20]; but that may not be typical of intercluster space [19]. The 0.3 μG figure corresponds to $A \approx 4 \times 10^{13}$ T m. Cosmological fields over the size of the known universe, $\approx 1.5 \times 10^{26}$ m, are not known, but are thought to have an upper limit [18] of $(0.2–1) \times 10^{-9}$ G, corresponding to $A < 10^{17}$ T m.

Experimental results disclose no reproducible signal above the noise, hence, the product of potential and inverse Compton length squared is $A\mu_\gamma^2 < 2 \times 10^{-9}$ T m/m^2; so, if $A \approx 10^{12}$ T m due to cluster level

fields, $\mu_\gamma^{-1} > 2 \times 10^{10}$ m. As for comparison with other experiments, the figure based on cluster fields is a factor of >40 larger, hence, m_γ smaller, than the best limit obtained via study of the Jovian magnetic field. The best data from true tabletop experiments [2,6] of deviations from Coulomb's law give $\mu_\gamma^{-1} > 3 \times 10^7$ m. The present results are a factor $\approx 6 \times 10^2$ more sensitive. A tabletop limit based on a low-temperature null test [21] of Ampère's law gives $\mu_\gamma^{-1} > 4 \times 10^5$ m. The present results are a factor $>10^4$ more sensitive. We are in a galaxy, and potentials from cluster or intercluster fields might be partly neutralized by galactic potentials. If we, then, conservatively use galactic fields prior to a reversal, $\mu_\gamma^{-1} > 1 \times 10^9$ m, a figure still superior to that derived from the Jovian field. If, however, $A \approx 4 \times 10^{13}$ T m (due to the lower value of intercluster filament fields), then $\mu_\gamma^{-1} > 2 \times 10^{11}$ m.

The present simple apparatus is more sensitive than prior experiments because it makes use of estimates of large-scale cosmic magnetic fields, which are associated with very large vector potentials. Since the presence of photon mass introduces a length scale into electromagnetism, detection requires interrogation of large volumes or extraordinary precision in a small-scale experiment. In contrast to purely astrophysical arguments regarding photon mass, no assumption is needed in the present approach regarding how the large-scale fields are generated. The experiment gives a limit on $A\mu_\gamma^2$, rather than μ_γ itself, because values of A inferred from astrophysical fields are uncertain. If our basic method were made sufficiently sensitive to make use of a calibrated source of A, then it would yield a value or limit for μ_γ. Improvements in sensitivity of the present method can be achieved by conducting the experiments in a quieter location, use of a larger toroid, and by improving estimates of the ambient vector potential based on better mapping of cosmic magnetic fields. It may also be possible to evaluate a bound on the source term $\mu_\gamma^2 A$ by independently measuring (with allowance for the possibility of photon mass) true currents and magnetic fields in the solar system. Finally, the energy in the potentials, if $\mu_\gamma > 0$, gives rise to torque even though the Lorentz force law [2] is unchanged in the presence of photon mass; therefore, the concept presented here may bear upon other short-range forces.

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