Simulations of thermoelastic triangular cell lattices with bonded joints by finite element analysis

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June 5, 2017

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Keywords (bi-material, lattices, thermal expansion, curved rib, welded joints, bonded joints, finite element simulation)

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Preprint **Ha, C. S., Plesha, M. E., Lakes, R. S., "Simulations of thermoelastic triangular cell lattices with bonded joints by finite element analysis", Extreme Mechanics Letters, 12, 101-107 (2017).**

Abstract

Thermoelastic triangular cell lattices composed of bi-material curved ribs were designed and analyzed by finite element simulation. Positive, negative, or zero thermal expansion was possible by varying rib curvature if joints can pivot freely, as expected. Welded or bonded joints result in nonzero expansion but smaller in magnitude than that of a constituent material having higher thermal expansion coefficient. The effects of rib curvature variation for bonded joints were found to be negligible. We present a square lattice with bonded joints that has zero net thermal expansion; each curved bi-material rib has zero expansion.

1 Introduction

Various engineering structures in fields such as aerospace, civil engineering, and microelectronics often undergo large temperature changes [1]. They lead to thermal stresses caused by different thermal expansions in components of a structure which are made of materials with dissimilar coefficients of thermal expansion. In the field of aerospace, supersonic and hypersonic vehicles exhibit significant thermal stresses, and thermodynamic propulsion plants also experience similar stresses [2]. For these structures, dimensional stability (*i.e.*, structural integrity) is surely a key consideration. An exmaple of the importance of dimensionally stable design of structures is the Hubble space telescope; considerable thermal distortions were produced by rapid temperature changes during its orbit, which led to undesired vibration of the telescope and the arrays [3]. A material's coefficient of thermal expansion is thus clearly one of the driving factors when selecting materials for structures subject to large fluctuations in temperature [4].

Materials of zero or minimal thermal expansion can provide dimensionally stable designs of structures subject to large fluctuations in temperature. For example, zero or nearly zero thermal expansion is desirable in fields in which precise positioning of parts is critical, such as optics and electronics [5]. Similarly, aerospace and civil engineering applications, like piping systems designed with tight dimensional tolerances, are required to have minimal or zero thermal expansion for achieving dimensional stability under extreme variation in temperature [6]. By contrast, materials exhibiting negative thermal expansion are of interest for applications need to contract with increases in temperature [7]. It is also possible to control any desired thermal expansions (such as zero, or large positive, or large negative) in composite materials with void spaces by tuning design of their microstructure [8] [9].

Recently, Lehman and Lakes [10] showed that zero coefficient of thermal expansion in a lattice made of bi-material curved rib elements can be achieved by designing hierarchical material structures with carefully chosen geometry and materials while optimizing the total mechanical stiffness of an equilateral triangular lattice, as shown in Figure 8. Two different metals with different positive thermal expansion coefficients were used to design this microstructure, which is composed of ribs whose cross section was rectangular. The difference in thermal expansion coefficients leads to bending of the rib during temperature change, which results in a decrease of the distance between its ends which is exactly conterbalanced by overall thermal expansion of the rib. By carefully tuning geometric parameters of the bi-material curved rib, zero net thermal expansion in a honeycomb or lattice structure can be achieved. An overall thermal expansion coefficient for an individual bi-material curved rib element with pin-ended joints, as depicted in Figure 2, is provided by Equation 1 [10]. The derivation of this equation was based on Timoshenko's work for bi-material strips [11]

$$\begin{aligned}\alpha_{net} = (\alpha_1 - \alpha_2)\frac{L_{arc}}{t}\Big(\frac{\theta}{12}\Big)\frac{6(1+m^2)}{3(1+m)^2 + (1+mn)(m^2 + \frac{1}{mn})} \\ + \ \frac{\alpha_1 + \alpha_2}{2} \ + \ (\alpha_2 - \alpha_1)\Big[\frac{4m^2 + 3m + \frac{1}{mn}}{nm^3 + 4m^2 + 6m + \frac{1}{mn} + 4} - \frac{1}{2}\Big]\end{aligned} \tag{1}$$

where α_1 and α_2 are the thermal expansion coefficients of material one and two, respectively, E_1 and E_2 are the elastic modulus of material one and two, respectively, L_{arc} is the arc length of the curved rib, m is the thickness ratio of material one to material two (*i.e.*, $m = \frac{a_1}{a_2}$), n is the elastic modulus ratio of material one to material two (*i.e.*, $n = \frac{E_1}{E_2}$), θ is the included angle and t is the total thickness of the rib. In order to obtain zero coefficient of thermal expansion, material one, positioned on the inner portion of the curved rib, is required to have a smaller thermal expansion coefficient than material two on the outer portion. Invar was used as material one, while material two was steel, and their material properties are given in Table 1.

Table 1: Material properties of selected materials.

	Material 1 (invar)	**Material 2 (steel)**
Elastic Modulus, E	140 GPa	200 GPa
CTE, α	1 μstrain/K	12 μstrain/K
Poissons Ratio, ν_{LT}	0.28	0.3

The analysis of such a lattice [10] relies on pin-ended joints between the bi-material curved ribs. These joints allow a force transmission to adjacent ribs while rotation is unconstrained; the ends of adjoining ribs are free to rotate with respect to one another. For these reasons, the curvature, and hence the moment, throughout a rib is uniform, with the result that it was straightforward to obtain the analytic solution given in Equation 1. For other engineering reasons such as limitations during the manufacturing process, bonded (or welded) joints between these ribs may be desirable or may be necessary. For example, we are currently investigating the use of 3-D printing to fabricate new materials, and this technology will most likely require bonded joints. With bonded joints, the curvature and moment will not be uniform and the resulting differential equations will be difficult or impossible to solve in closed form. Hence, the use of finite element analysis is effective.

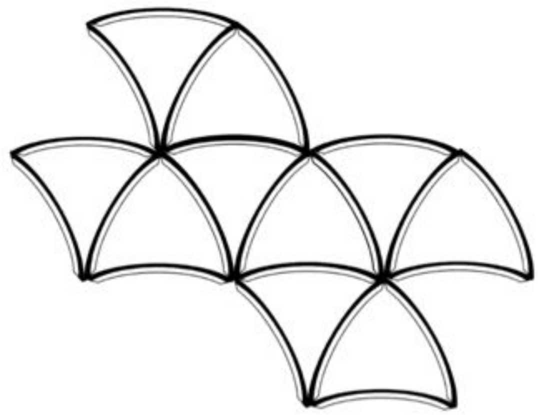
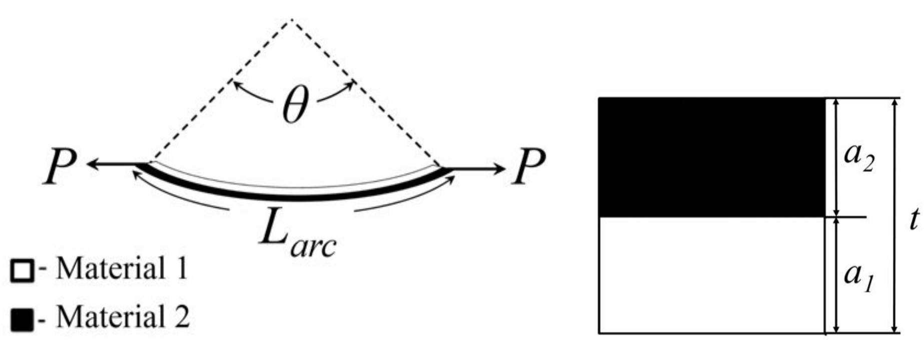


Figure 1: The equilateral triangular lattice composed of bi-material curved ribs [10]. Material one with a lower coefficient of thermal expansion is shown as white (on the inner portion of each rib), while material two with a higher coefficient of thermal expansion is shown as black (on the other portion of each rib).



(a)

(b)

Figure 2: (a) The loading state used to determine analytical results of zero coefficient of thermal expansion on the bi-material curved rib for pin-ended joints [10]. (b) L_{arc} is the arc length of the rib, θ is the included angle, and P represents axially applied load. The cross-sectional area of the bi-material curved rib [10] is shown.

In the present manuscript, an individual bi-material curved rib connected by pin-ended joints was modeled using the commercial finite element program ANSYS to verify the analytical results of reference [10]. A finite element model of the bi-material curved rib with bonded joints was then created and studied to determine if it could achieve zero or reduced thermal expansion. For both ribs, the effective coefficients of thermal expansion were calculated. The effects of a variation of the rib curvature for the rib with bonded joints were also investigated. To incorporate the effects of interactions between adjacent ribs in a lattice, several finite element models of an equilateral triangular lattice were created by assembling the finite element models representing each of individual bi-material curved ribs. The influence of bonded joints between such ribs in the lattice was then investigated subject to uniform temperature change. The effective thermal expansion coefficient was also computed. A change in the orientation of some of the ribs in the lattice was studied to observe how such a change would influence the overall thermal expansion. Finally, we conclude this paper with discussion of a square lattice with bonded joints that has zero net thermal expansion provided that each rib has zero thermal expansion.

2 An Individual Bi-material Curved Rib

2.1 Determination of Optimized Geometric Parameters

In order to achieve zero thermal expansion coefficient of an individual bi-material curved rib, geometric parameters and material properties need to be specified carefully [10]. With Equation 1 describing the net thermal expansion coefficient of the rib in terms of geometric parameters and elastic moduli of the two materials, the included angle to obtain a zero net thermal expansion coefficient can be numerically computed by varying the invar fraction, as illustrated in Figure 3. Invar fraction is the ratio of the thickness of invar to the total thickness of the rib, and the rib aspect ratio, AR, is the ratio of the arc length to the total thickness of the rib. Each curve denotes a different aspect ratio for representing slenderness of the rib. Materials used in this graph are typical invar and steel. The prior analytical results described an optimum invar fraction as approximately 45 % [10]. The input to FEM requires a more precise value than is needed to draw graphs for an analytical result. This value was extracted from Equation 1 to show that the optimum fraction was actually 46.39 % regardless of the rib aspect ratio. This parameter was confirmed by obtaining desired thermal expansion of a rib with pin-ended joints by finite element analyses.

The optimum invar fraction of 46.39 % was then substituted into Equation 1 to obtain optimized geometric parameters of an individual bi-material curved rib. With the total thickness of the rib of 1 mm and the rib aspect ratio of 10, the included angle was calculated as 0.4909 radians, the radius of curvature was found to be 20.3684 mm, and the thickness of materials one and two was computed as 0.4639 mm and 0.5361 mm, respectively. These optimized values are summarized in Table 2.

Table 2: Optimized geometric parameters of an individual bi-material curved rib.

Optimized geometric parameters	**Values**
The thickness of the material 1, a_1	0.4639 [mm]
The thickness of the material 2, a_2	0.5361 [mm]
The included angle, θ	0.4909 [radians]
The radius of curvature, ρ	20.3684 [mm]

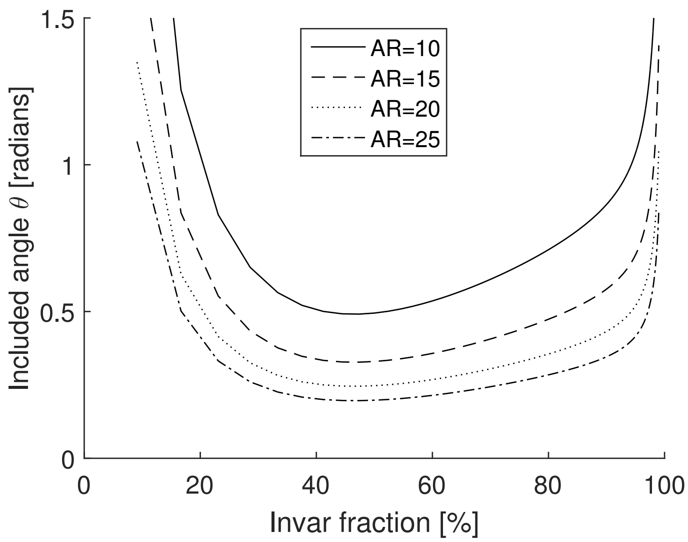


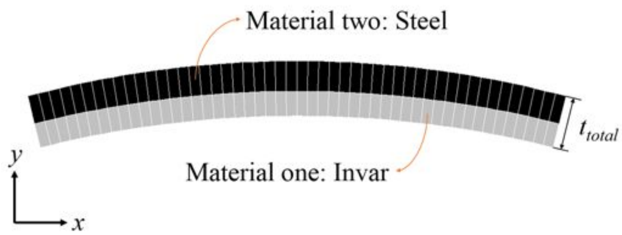
Figure 3: The included angle of the bi-material curved rib versus the invar fraction with various aspect ratios to achieve zero thermal expansion coefficient. For all aspect ratios, the minimum included angle θ, which corresponds to the rib with the least curved geometry, occurs for the invar fraction of approximately 46.39 %.

2.2 Design of an Individual Bi-material Curved Rib

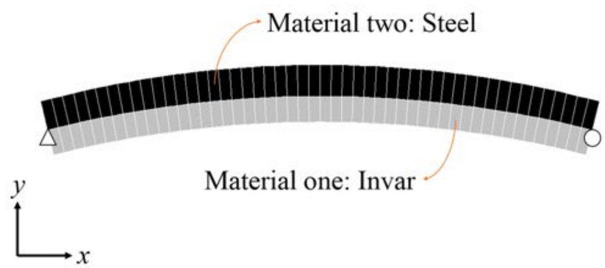
2.2.1 Baseline Finite Element Model

A baseline finite element model of an individual bi-material curved rib, as depicted in Figure 4a, was created by using the optimized geometric parameters given in Table 2. This model allows simulation of both the pin-ended and the bonded joints by imposing two different sets of support conditions. For design of the baseline model, PLANE 183 elements were used in ANSYS with linear elastic isotropic plane stress behavior. The width in the direction perpendicular to xy-plane was 1 mm. This element is a higher order two-dimensional 8-node element, and has a quadratic displacement behavior and two degrees of freedom (d.o.f.) at each node: nodal translation in the x and y directions. Material one positioned on the inner portion of the rib was invar, while material two on the outer portion was steel. The material properties given in Table 1 for invar and steel were used for this model.

In general, an increase in the number of finite elements in a model produces greater accuracy of the results at the expense of greater computation time. A convergence analysis was performed to determine a suitable number of elements in order to have acceptable accuracy. The model meshed with 100 elements per rib was considered as reference. It was found that at least 80 elements per rib were required to obtain displacement error less than 1 %. For this reason, unless specified otherwise throughout this paper, all of the finite element models are meshed using a total of 100 elements per rib.



(a)



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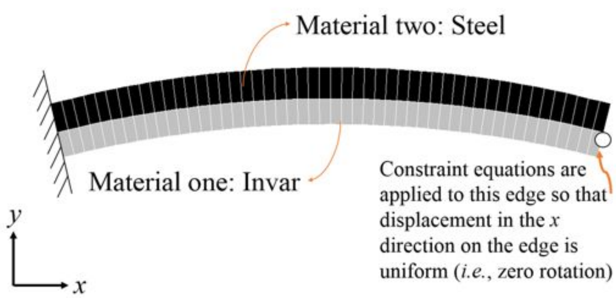


(c)

Figure 4: (a) A two-dimensional baseline finite element model of an individual bi-material curved rib. PLANE 183 elements in ANSYS were used with plane stress behavior. Geometric parameters listed in Table 2 were applied to this model. The width was set to 1 mm in the direction perpendicular to xy-plane. All nodes and elements lie in the plane of the figure. (b) A two-dimensional finite element model describing a simply supported bi-material curved rib. (c) A two-dimensional finite element model describing an individual bi-material curved rib with bonded joints.

2.2.2 Finite Element Model of an Individual Bi-material Curved Rib with Pin-ended Joints

The finite element model shown in Figure 4b was created so that it could reproduce the same support conditions that were employed in the analytical study [10]. Namely, this model is simply supported with a pin at the left and a roller at the right that constrains the vertical motion at the right tip. Hence, this model is appropriate to verify the prior study of a zero net thermal expansion of an individual bi-material curved rib with pin connections at ends [10]. Since the support conditions do not constrain rotation at either end, both ends are allowed to rotate freely, and the right end is free to displace horizontally due to either an axial load at the right tip or a temperature change. Note that the supports (*i.e.*, locations of the pin and the roller) are positioned at the interface between materials.

2.2.3 Finite Element Model of an Individual Bi-material Curved Rib with Bonded Joints

A finite element model representing an individual bi-material curved rib with bonded joints on both ends was developed, as shown in Figure 4c. This model is built-in at the left and is supported with a roller at the right tip with constraint equations that prevent rotation of the right end while allowing a uniform translation of the right end in the x direction (*i.e.*, nodal displacements of the right end in the x direction are uniform). Hence, these support conditions represent bonded joints at both ends of the curved rib. Note that the roller support is located at the interface between materials on the right edge.

2.3 Results and Discussion

2.3.1 Thermal Expansion of an Individual Bi-material Curved Rib with Bonded Joints

Uniform temperature changes, ΔT, of 1K, 10K, 20K, 50K, and 100K were applied to both finite element models with pin-ended and bonded joints. Since our models are linear elastic, only one temperature change would be adequate to characterize the thermal expansion behavior, but we believe the usefulness of the results is enhanced by considering multiple temperature changes. Note that the finite element model with the pin-ended joints was used to numerically study the validity and/or limitations of the analytical results [10] regarding parameters that allow design of a rib with a zero net thermal expansion. Thus, this model was expected to have zero thermal expansion. For useful comparisons, additional finite element models for a homogeneous curved rib made entirely of invar or entirely of steel were also modeled, and the corresponding thermal expansions were computed numerically for both the pin-ended and the bonded joints.

Figure 5 demonstrates numerically obtained horizontal tip displacements for both homogeneous and bi-material curved ribs as a function of uniform temperature change. In this graph, solid data points represent the ribs with the bonded joints, while the ribs that are pin connected are illustrated as hollow data points. As expected, for homogeneous ribs, there was very small difference in horizontal tip displacements regardless of the joint conditions; these ribs produced uniform thermal expansion which resulted in no cross section rotation of the ends, and the coefficients of thermal expansion were about 1 μstrain/K for a rib made of invar and about 12 μstrain/K for a rib made of steel. This is consistent with the notion that constraint of rotation should not be influenced by uniform deformation.

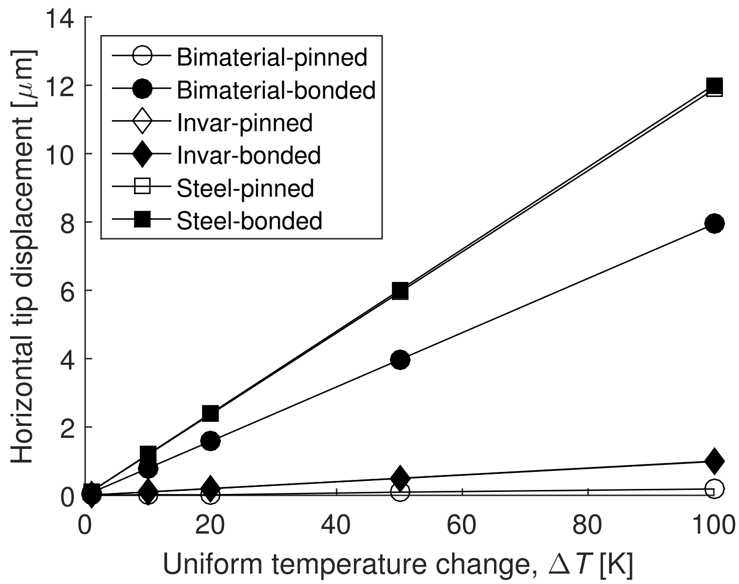


Figure 5: The horizontal tip displacement of homogeneous and bi-material curved ribs versus uniform temperature change, ΔT.

A bi-material curved rib with pin-ended joints produced nearly zero horizontal tip displacements. The corresponding coefficient of thermal expansion was found to be 0.1884 μstrain/K. Therefore it can be concluded that there is good agreement between the finite element model and the analytical results [10]. On the other hand, a bi-material curved rib with bonded joints showed a substantial increase in the horizontal tip displacement, and the thermal expansion coefficient of this rib was computed to be 8.0157 μstrain/K. This rib experienced a horizontal tip displacement that is 42.5 times larger than that for the pin-ended joints subjected to the same temperature change, but still exhibited approximately 33 % less expansion than that of a homogenous rib made of steel.

The constraint of rotation reduces the bending effect that would otherwise enable the bi-material curved rib to achieve zero expansion. As a result, it appears that the optimized geometric parameters to obtain zero thermal expansion according to the analytical results [10] need to be revised for a rib with bonded joints.

2.3.2 The Effects of Rib Curvature

A further investigation on a bi-material curved rib with bonded joints was performed to determine whether careful tuning of the geometric parameters could result in zero net thermal expansion. Four rib curvatures were generated by varying the radius of curvature, ρ, and the included angle, θ, simultaneously while holding the distance between the ends of the rib constant, as shown in Figure 6. The thick solid curve shown represents the rib curvature obtained using the optimized geometric parameters (ρ_{opt} and θ_{opt}) given in Table 2. Geometric parameters for other rib curvatures are given in Table 3.

The baseline finite element model of the bi-material curved rib was modified in order to develop finite element models of the bi-material curved rib having rib curvatures shown in Figure 6. Similar to the analysis in the previous section, horizontal tip displacements of these models due to uniform temperature changes were computed numerically and are illustrated in Figure 7. In addition to the

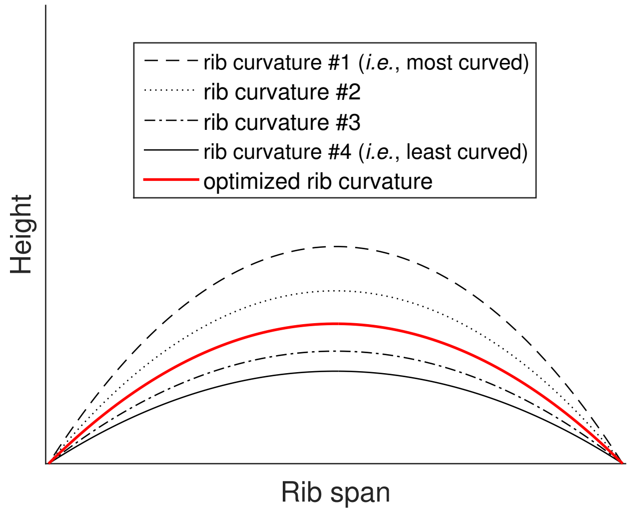


Figure 6: Several rib curvatures , shown to scale, for the bi-material curved rib. Each curve was generated by varying the optimized geometric parameters while holding the distance between the ends of the rib constant. The thick solid curve shown represents the optimized curvature of the bi-material curved rib [10].

Table 3: Geometric parameters for several rib curvatures. The radius of rib curvature and the included angle were varied simultaneously while holding the distance between the ends of the rib constant.

	ρ/ρ_{opt}	θ/θ_{opt}
Rib curvature #1	0.685	1.541
Rib curvature #2	0.816	1.232
Rib curvature #3	1.237	0.806
Rib curvature #4	1.5	0.663

ribs with bonded joints, pin-ended ribs associated with each rib curvature shown in Figure 6 were also studied to observe how they would result in thermal expansion depending on varying the rib curvature under uniform temperature change. As before, hollow data points illustrate the ribs with pin-ended joints, while solid data points represent the ribs with bonded joints.

As depicted in Figure 7, the horizontal tip displacements of the bi-material curved ribs with pin-ended joints decrease as the rib becomes more curved (*i.e.*, increasing rib curvature). If the rib is more curved than one designed based on optimal parameters leading to zero thermal expansion, material two with a higher coefficient of thermal expansion (*i.e.*, the outer portion of the rib) dominates deformation, which leads to negative thermal expansion. On the other hand, material one with a lower coefficient of thermal expansion plays a substantial role resulting in positive thermal expansion when the rib is less curved. This indicates that such ribs allow one to produce either positive or negative thermal expansion by varying rib curvature (*i.e.*, changing ρ and θ simultaneously) while holding other geometric parameters constant.

In contrast, for the ribs with bonded joints, the horizontal tip displacements were not altered significantly regardless of how much these ribs were curved. The thermal expansion coefficients

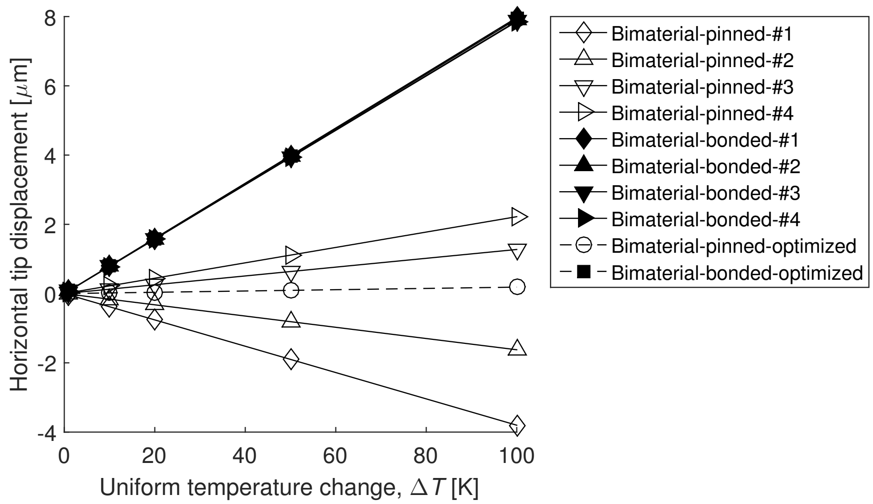


Figure 7: The horizontal tip displacement versus uniform temperature change. Each line represents the horizontal tip displacement of the bi-material curved ribs for several rib curvatures described in Figure 6 with either pin-ended or bonded joints. While the ribs with the pin-ended joints can produce either positive or negative thermal expansion by controlling geometric parameters, such thermal expansions are not possible with the ribs with the bonded joints.

for these ribs were found to be approximately 8 μstrain/K. A study of the effect of rib curvature was done by holding a distance between the ends of the rib while varying the radius of curvature, ρ, the included angle, θ, and the arc length, $\L_{arc}$. For the ribs with bonded joints, a generated bending moment varies depending on the rib curvature. As the rib curvature increases, the arc length extends, which minimizes the effect of the bending moment. This results in the minimal influence on thermal expansion. Consequently, it would be desirable to have an analytical solution for a rib with bonded joints. This would help to explore the design space to determine if and how zero thermal expansion could be obtained. However, even with such a solution, it would not provide the same results as for a rib with bonded connections that is part of a lattice, because of the effects of rotation of the ends.

2.3.3 The Effects of Rib Aspect Ratio

If ribs are straight and slender, it is known that a lattice with bonded joints behaves similarly to a lattice with pin joints. A difference is observed in the present lattice which has curved ribs. The reason is that curved ribs when deformed tend to rotate at the end. A bending moment is generated at a bonded joint but not at a pin joint. If ribs are straight there is neither a tendency to rotate nor a bending moment.

A brief study of the effects of aspect ratio, AR, was carried out. As defined earlier, the aspect ratio is the ratio of the rib's arc length to its total thickness; increasing AR corresponds to an increasingly slender rib. Although we do not present detailed results here, we found that under uniform temperature change, a rib with pin-ended joints achieves nearly zero thermal expansion regardless of aspect ratio whereas thermal expansion of a rib with bonded joints increases as the aspect ratio (slenderness) increases. Therefore the role of bending moments in ribs bonded at the joints persists even for slender ribs.

3 Equilateral Triangular Lattices Composed of Bi-material Curved Ribs

In Section 2 of this paper, finite element models were developed to represent individual bi-material curved ribs with pin-ended and bonded joints. Although these finite element models have provided insightful results, they do not necessarily characterize the thermal expansion behavior of a lattice that is composed of these ribs. Thus, it is necessary to develop finite element models of possible lattice structures, and several of these are considered in this section and the next.

3.1 Design of an Equilateral Triangular Bi-material Lattice

3.1.1 A Finite Element Model of an Equilateral Triangular Bi-material Lattice

A finite element model of a two-dimensional equilateral triangular lattice composed of bi-material curved ribs was created by assembling the finite element model of an individual bi-material curved rib, as shown in Figure 8a. The elastic moduli and thermal expansion coefficients for invar and steel described in Table 1 were applied to the model. Thus, this model is adequate to represent the lattice proposed by reference [10]. Node numbers at the locations of joints are shown in this figure. This model is supported by a pin at node 1 (*i.e.*, the left-bottom node) and rollers at nodes 2, 3, 4, and 5 (*i.e.*, the remaining bottom nodes). Additionally, in order to reproduce bonded joints between the ribs in the lattice, a rotational degree of freedom (*i.e.*, rotation about the z direction) on edges at both ends of the ribs needs to be constrained in such a way that adjacent edges of adjoining ribs have the same rotational displacement. To do so, each of these edges was meshed with two BEAM 189 elements in ANSYS, as shown in Figures 9a and 9b. This element is a three-dimensional quadratic three-node beam element having six degrees of freedom at each node; three translations in the x, y, and z directions and rotations about the x, y, and z directions. To render these beam elements as two-dimensional, the superfluous degrees of freedom for all nodes of the beam elements were constrained to have zero values (*i.e.*, for each node of the beam elements, translation in the z direction and rotations about the x and y axes were given zero prescribed values). Moreover, a stiffer elastic modulus (100 times stiffer than steel) and the same rectangular cross sectional area were applied to the beam elements compared to that of the baseline model of an individual bi-material curved rib, for representing a realistic bonded joint in practice. A thermal expansion coefficient of these beam elements used at the ends was also set to zero.

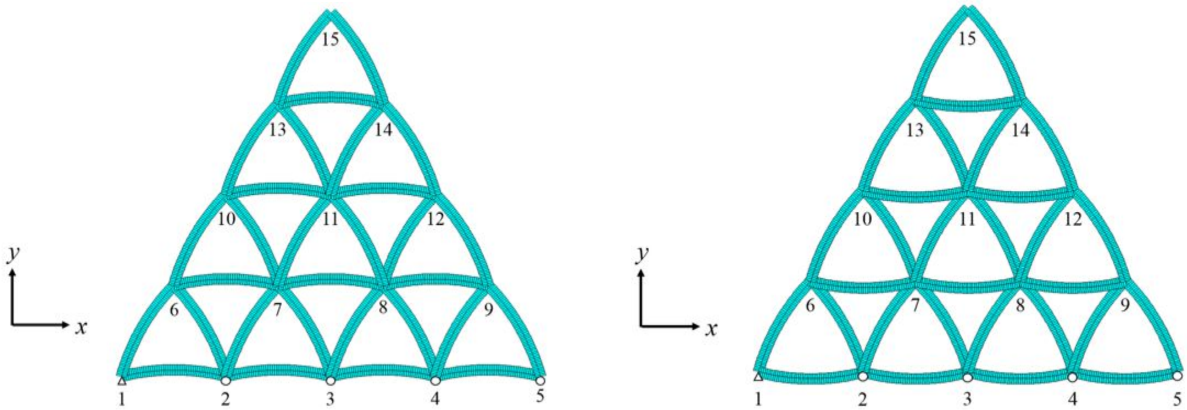
3.1.2 A Finite Element Model of an Equilateral Triangular Bi-material Lattice with Reversed Rib Curvatures

Figure 8b shows a finite element model representing a two-dimensional equilateral triangular lattice where the bottom ribs of each cell in the lattice are reversed compared to Figure 8a. Material properties, element types used for modeling, nodal locations, and support conditions are identical to the finite element model of the lattice described in the previous section.

3.2 Results and Discussion

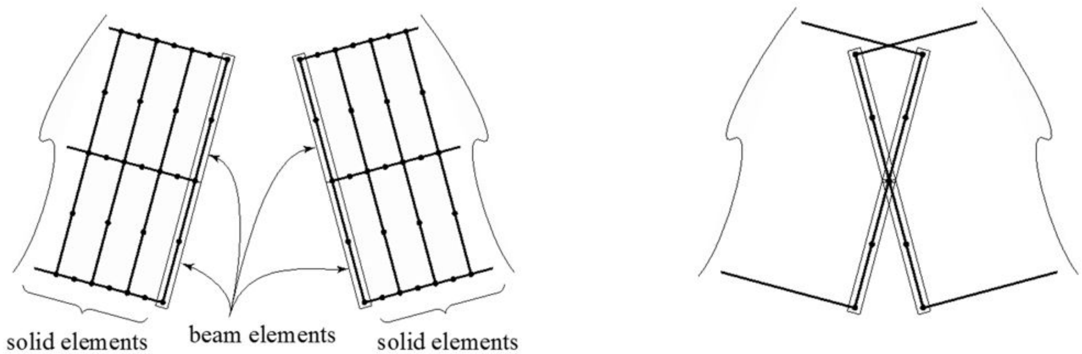
3.2.1 Thermal Expansion of an Equilateral Triangular Bi-material Lattice

Uniform temperature changes, ΔT, of 1K, 10K, 20K, 50K, and 100K were considered. Figure 10 illustrates overall thermal expansions in the x and y directions of an equilateral triangular bi-material lattice with two different joint conditions (*i.e.*, pin-ended and bonded joints), respectively.



(a) (b)

Figure 8: (a) A two-dimensional equilateral triangular lattice composed of bi-material curved ribs. Node numbers at the locations of joints are shown. (b) A the two-dimensional equilateral triangular lattice composed bi-material curved ribs with reversed rib curvatures. This model is identical to that described in Figure 8a except the bottom ribs of each cell have been reversed. Note that node numbers are shown where the joints are located.



(a) (b)

Figure 9: A representation of bonded joints in the lattice model using four 3-node beam elements (BEAM 183) on the ends of ribs. Two ribs are shown separated for clarity. (a) The rib has two stiff beam elements, and similarly for the rib on the right. (b) The nodes and element edges for the solid elements are not shown for clarity. The beam elements share a common node, and due to their high stiffness, they are essentially rigid and effectively model a bonded connection between the two ribs.

Solid data points represent the lattices with the bonded joints while the pin-ended lattices are given as hollow data points. Such displacements of homogeneous lattices made entirely of invar or entirely of steel were also plotted in these figures for useful comparisons. The results showed that the overall thermal expansion displacements of the bi-material lattice with the bonded joints were significantly larger in both x and y directions than those with the pin-ended joints; the corresponding effective coefficients of thermal expansion were calculated to be approximately 6.6707 μstrain/K for the bonded lattice and -0.0326 μstrain/K for the pin-ended lattice.

As a result, a pin-ended bi-material lattice allows tuning of thermal expansion coefficient to approach zero, while a similar lattice with bonded joints exhibits substantial thermal expansion. Moreover, it is worthwhile noting that the pin-ended bi-material lattice produces very small negative coefficient of thermal expansion rather than small positive value that has been observed in a study of an individual bi-material curved rib with pin-ended joints. Thus, an analytical solution for a lattice with bonded joints would be desirable, which would help to study the design space to determine whether zero thermal expansion could be obtained.

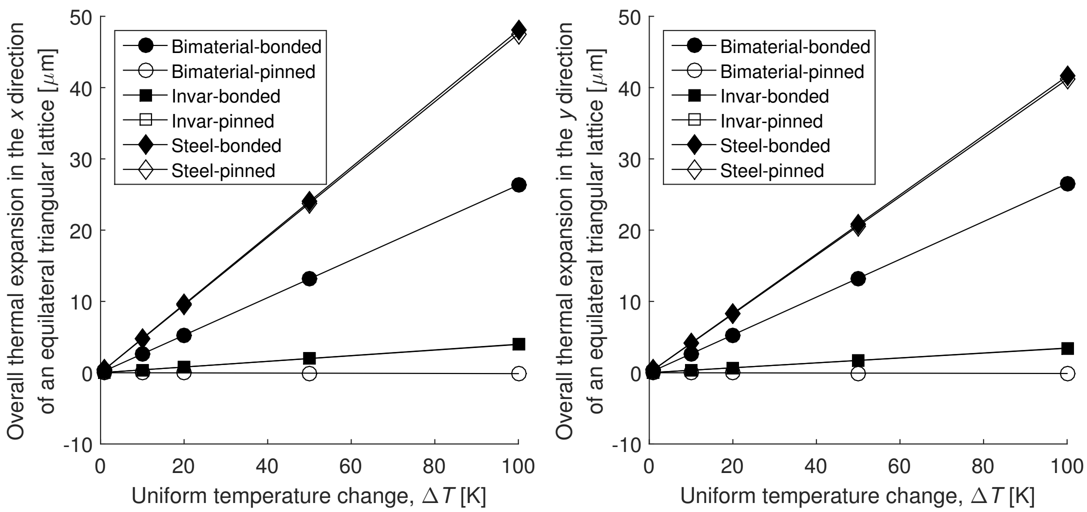


Figure 10: Overall thermal expansions in the x and y direction of an equilateral triangular lattice composed of bi-material curved ribs with two different joint conditions.

3.2.2 Thermal Expansion of an Equilateral Triangular Bi-material Lattice with Reversed Rib Curvatures

Figure 11 illustrates overall thermal expansions in the x and y directions of an equilateral triangular bi-material lattice having reversed curvature bottom ribs of each cell (shown in Figure 8b) with both pin-ended and bonded joints due to uniform temperature changes. A change in the orientation of the rib curvature from concave-down to concave-up had negligible effect on the overall thermal expansion in a pin-ended lattice regardless of its material constitution. On the other hand, such change resulted in an overall thermal expansion of a lattice with bonded joints that is larger in the x direction by a factor of 1.20 and in the y direction by a factor of 1.03, when compared to a similar lattice with no reversed rib curvatures. Therefore, it appears that thermal expansion of a bi-material lattice with bonded joints is sensitive to the shape of each cell.

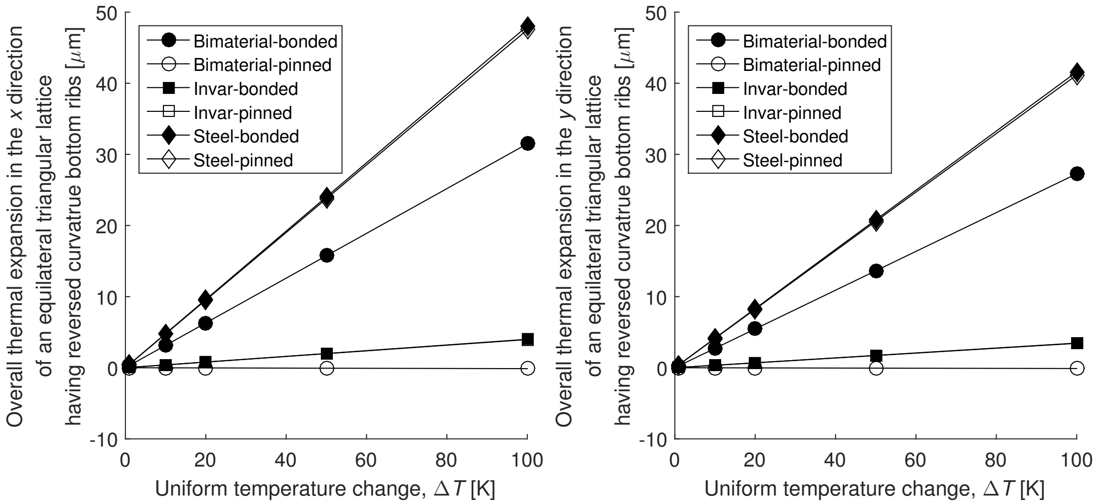


Figure 11: Overall thermal expansions in the x and y direction of an equilateral triangular lattice having reversed curvature bottom ribs of each cell composed of bi-material curved ribs with two different joint conditions.

4 A Square lattice composed of bi-material curved ribs.

Zero thermal expansion in lattices with bonded joints is possible if ribs are orientated with cubic symmetry, as shown in Figure 12. This lattice is stiff in the principal directions but not in oblique directions. Due to a uniform temperature change, each rib produce the same displacement and rotation. However, the joint rotation of each rib is accommodated by equal rotation of adjoining ribs, hence there is zero moment in each rib at each joint, even though joints are bonded. This leads to zero net thermal expansion provided that each rib individually has zero net thermal expansion and provided the temperature change is uniform. Due to a temperature gradient, this lattice will likely have thermal expansion, although perhaps this will be small. Further finite element studies are needed to elucidate this behavior.

5 Conclusions

This manuscript shows that an individual bi-material curved rib with pin-ended joints can exhibit desired thermal expansion, as expected. On the other hand, a rib with bonded joints produced substantial positive thermal expansion. The effects of rib curvature variation were negligible on thermal expansion of a rib with bonded joints, while a rib with pin-ended joints depended highly on it. The effective coefficient of thermal expansion of a rib with both pin-ended and bonded joints stays the same regardless of different slenderness. The role of bending moments in ribs bonded at the joints persists even for slender ribs. Similar results were observed for equilateral triangular lattices with these two different joint conditions. A change in the orientation of some of the ribs in a lattice had a negligible thermal expansion change in pin-ended lattices regardless of its material constitution. However, for lattices with bonded joints, the shape of cells in the lattice strongly influence the overall thermal expansion. A square lattice composed of bi-material curved ribs with bonded joints can provide zero expansion.

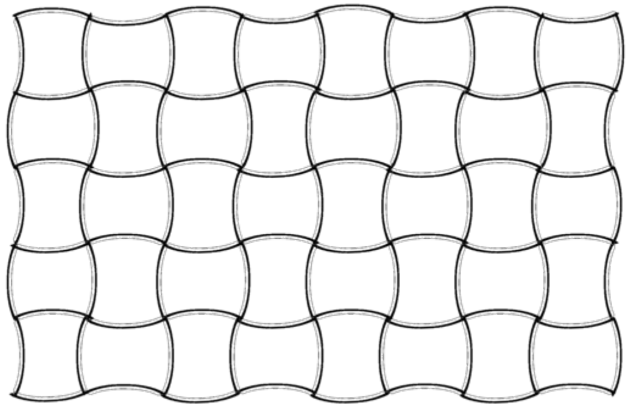


Figure 12: A square lattice composed of bi-material curved ribs with bonded joints. This lattice will display zero thermal expansion provided each rib has zero thermal expansion and the temperature change is uniform.

Acknowledgements The work presented in this paper was supported by the Defense Advanced Research Projects Agency-Lawrence Livermore National Laboratory (DARPA-LLNL) under the directorship of Dr. Judah Goldwasser.

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