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EXPERIMENTAL STUDY OF MICROPOLAR AND COUPLE STRESS ELASTICITY IN COMPACT BONE IN BENDING*

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Abstract—Generalized continuum theories such as couple stress theory and micropolar theory have degrees of freedom in addition to those of classical elasticity. Such theories are thought to be applicable to materials with a fibrous or granular structure. In this study we observe size effects in quasistatic bending of compact bone. The effects are consistent with micropolar theory. From them we evaluate two nonclassical elastic constants.

INTRODUCTION

In the development of the classical theory of elasticity it is assumed that the interaction upon a differential element of surface is specified completely by a force vector. If one assumes that in addition a couple vector may act on the surface element, one may develop the concept of couple stress, or distributed couple per unit area. The idea of couple stress was introduced by Voigt in 1887 and was used by the Cosserat brothers (1909) in the development of a generalized theory for mechanical behavior, which admits an asymmetric force–stress tensor. This theoretical development remained dormant until the 1960s during which various workers developed generalized continuum theories. Theories of elasticity with microstructure (Mindlin, 1964) and of microelastic solids (Eringen and Suhubi, 1964) are quite general and can describe various wave dispersion effects and resonances of the 'unit cell'. Contained as a special case within these is the Cosserat theory (Mindlin, 1965) which is considered identical to the micropolar theory of Eringen (1970). In this theory *points* within the material have the rotational degrees of freedom of a rigid body as well as the usual translational degrees of freedom. Cosserat theory contains as a special case the couple stress theory of Mindlin and Tiersten (1962) and of Toupin (1962). Finally, classical elasticity is contained as a special case within couple stress elasticity.

Many authors then solved boundary value problems under these theories. Of particular interest is the effect of couple stress in the problem of finding the stress concentration around a hole. Stress concentration factors are predicted to be smaller in a couple stress elastic material than in a classically elastic material. The reduction in stress concentration is most significant when the hole size approaches a certain characteristic length, which is an additional elastic constant in couple stress theory.

The question of the magnitude of couple stress elastic coefficients has been addressed by several investigators. For example Kröner (1963) points out that there must be a local resistance to curvature as required by couple stress theory since the interatomic forces exert influence further than one atomic spacing. In polycrystalline materials the structural element of interest is the constituent grain, and the characteristic length might be of the order of the grain size. The magnitude of couple stress elastic coefficients has been predicted for structured materials on the basis of continuum approximations in the case of a lattice of elastic beams (Askar and Cakmak, 1963), a laminated composite (Herrmann and Achenbach, 1967) and a honeycomb structure of cubical cells (Adomeit, 1967). Bone, a natural fibrous composite, has also been modelled in the light of extended continuum theory. It has been suggested that cancellous (Swenson *et al.*, 1979) and compact (Lakes, 1980) bone may obey couple stress theory. In such continuum descriptions of structured systems the characteristic length is generally predicted to be of the order of the size of the structural elements.

Few experimental efforts have been made to verify couple stress and related theories. Attempts to find evidence for couple stress elastic behavior in metals have not been successful (Schijve, 1966; Ellis and Smith, 1968). Quasistatic experiments performed by Gauthier and Jahsman (1975) on a composite model to observe micropolar elastic behavior, disclosed no evidence of such behavior. Optical studies of crystalline KNO_3, interpreted by Askar (1972) on the basis of micropolar theory suggest a characteristic length of the order of the lattice parameter, far too small to be seen in a macroscopic mechanical experiment.

Experiments by Perkins and Thomson (1973) on a foam material suggest couple stress elastic behavior, but artifacts due to viscoelastic behavior may have contributed. Yang and Lakes (1980) have reported evidence of couple stress elasticity in bone in quasistatic torsion. Lakes (1981) has performed dynamical experiments on bone, which disclose evidence of couple stress.

In this article we report the results of quasistatic bending experiments on human compact bone. The results are interpreted in terms of Mindlin and Tiersten's couple stress theory and Eringen's micropolar theory.

THEORY

Both the couple stress theory of Mindlin and Tiersten (1962) and the micropolar theory of Eringen (1970) will be discussed. A simple approximate solution for the bending problem is available in couple stress theory, and a relatively complicated exact solution is available in micropolar theory. Results of bending experiments will be evaluated in the light of both solutions.

The constitutive equations of isotropic couple stress theory are [2]:

$$t_{ij}^{sym} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} \qquad (1)$$

$$m_{ij}^{D} = 4\eta\kappa_{ij} + 4\eta'\kappa_{ji}, \qquad (2)$$

in which t^{sym} is the symmetric part of the force stress tensor, m^D is the deviator of the couple stress tensor, e_{ij} is the strain, κ_{ij} is the curvature, λ and μ are the Lamé constants, and η and η' are couple-stress elastic constants. The quantity $\ell = (\eta/\mu)^{1/2}$ has dimensions of length and is referred to as a characteristic length. The Mindlin–Tiersten couple stress theory is at times referred to as indeterminate couple stress theory since the antisymmetric part of the force-stress and the trace of the couple stress are not determined by the constitutive equations. These quantities depend on boundary tractions in specific situations.

The problem of bending of a beam with rectangular cross section has been examined by Koiter (1964). The flexural rigidity J is given by:

$$J \leqslant E\frac{bh^3}{12} + 4\mu\ell^2\left(1 + \nu^2 + 2\nu\frac{\eta'}{\eta}\right)bh, \qquad (3)$$

in which $E = \mu(3\lambda + 2\mu)/\lambda + \mu)$ is Young's modulus, $\nu = \lambda/2(\lambda + \mu)$ is Poisson's ratio, and b is the width and h the height of the cross section. The equality holds only for $\eta'/\eta = -\nu$. We have performed a bending analysis for a beam with circular cross section of diameter d, and have obtained similar results:

$$J \leqslant \frac{\pi E}{64}d^4 + \pi G\ell^2 d^2\left(1 + \nu^2 + 2\nu\frac{\eta'}{\eta}\right), \qquad (4)$$

in which again the equality holds only for $\eta'/\eta = -\nu$. We may write this

$$J \leqslant \frac{\pi E}{64}d^4 + \pi E\ell_{bl}^2 d^2 \qquad (5)$$

in which $\ell_{bl}^2 \equiv (G/E)\ell^2[1 + \nu^2 + 2\nu(\eta'/\eta)]$ is a characteristic length for bending. An expression of this sort is useful in the analysis of experimental results since it predicts a plot of rigidity/(size)^2 vs (size)^2 to be a straight line offset from the origin. The offset is related to the characteristic length, and is zero in a classically elastic solid. The situation is similar to the case of torsion, in which Koiter (1964) has obtained an exact solution which Yang and Lakes (1980) have used in the analysis of size effects in torsion experiments.

The constitutive equations of linear isotropic materials in the micropolar theory of Eringen (1968) are

$$t_{kl} = \lambda e_{rr}\delta_{kl} + (2\mu + \kappa)e_{kl} + \kappa e_{klm}(r_m - \phi_m) \quad (6)$$

$$m_{kl} = \alpha\phi_{r,r}\delta_{kl} + \beta\phi_{k,l} + \gamma\phi_{l,k} \qquad (7)$$

in which t is the force stress tensor, m is the couple stress tensor, e_{kl} is the usual small strain tensor, e_{klm} is the permutation symbol, r is the macrorotation vector, ϕ is the microrotation vector, λ and μ are the Lamé constants and κ, α, β and γ are additional elastic constants associated with micropolar theory. Observe that the Mindlin–Tiersten couple stress theory can be obtained as a special case of micropolar theory by imposing the restriction $\phi_k = r_k$. The observed shear modulus in the absence of microrotations is $\mu + \kappa/2$ (Cowin, 1970). Observe also that in micropolar theory the force stress and couple stress are completely determined by the constitutive equations.

The theory has been applied by Krishna Reddy and Venkatasubramanian (1979) to obtain an exact solution for the case of bending of a long circular cylinder of diameter $d = 2a$. The cylinder is subjected to couples M and $-M$ on its ends. The curvature $1/R$ and end moment M are related by the flexural rigidity $J = [M/(1/R)]$.

Analytically the flexural rigidity can be expressed by the form:

$$J = J_1\Omega = \frac{\pi d^4 E}{64} \times \left[1 + \frac{8N^2}{\nu + 1}\left(\frac{1 - (\beta/\gamma)^2}{(\delta_a)^2} + \frac{(\beta/\gamma + \nu)^2}{\zeta(\delta_a) + 8N^2(1 - \nu)}\right)\right] \qquad (8)$$

in which

$$\zeta(\delta_a) = (\delta_a)^2[(\delta_a I_0(\delta_a) - I_1(\delta_a))/((\delta_a I_0(\delta_a) - 2I_1(\delta_a))].$$

J_1 is the classical flexural rigidity, Ω is the ratio J/J_1. $N = \kappa/[2(\mu + \kappa)]$ is the coupling number, and

$$\delta^{-1} = \left[\frac{\gamma(\mu + \kappa)}{\kappa(2\mu + \kappa)}\right]^{1/2}$$

is a micropolar length parameter for bending. I_0 and I_1 are modified Bessel functions of order zero and one respectively and E is Young's modulus. Based on the

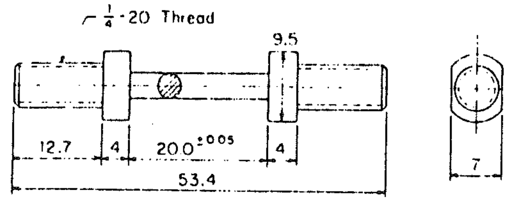
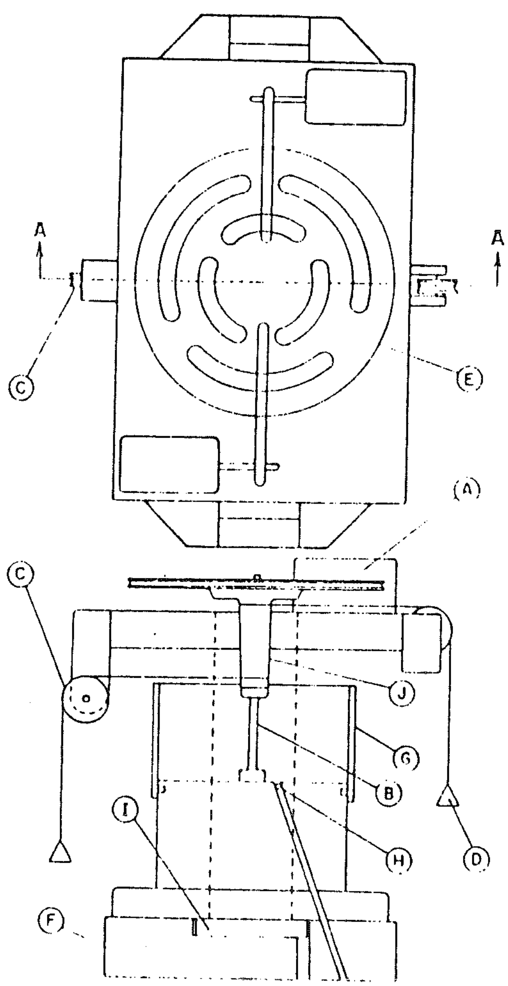


Fig. 1. Specimen configuration. Dimensions are in mm.

above result, one should be able to distinguish between micropolar elastic behavior and classical elastic behavior by measuring the flexural rigidity J of circular cylinders of different diameter $d = 2a$. The material coefficients E, N, β/γ, δ^{-1}, and ν can be obtained by curve fitting in a plot of J/d^2 vs d^2. In a classically elastic material, the micropolar quantities α, β, γ, and κ vanish, Ω becomes equal to unity, and the classical result for bending is recovered. The special case of couple stress elasticity is obtained by setting $N = 1$. This case is admissible on both thermodynamical and physical grounds (Cowin 1970) contrary to the earlier views of Kaloni and Ariman (1967).

Micropolar theory and its special case couple stress theory predict a stiffening effect which depends on the size of a specimen of material. This stiffening is predicted in the bending of plates (Gauthier and Jahsman, 1975) as well as beams and in the torsion of rods (Koiter, 1964; Gauthier and Jahsman, 1975). The stiffening effect becomes noticeable if the specimen diameter is ten times the characteristic length, and can become large as the specimen diameter approaches the characteristic length. Neither stiffening nor size effects are predicted in tension (Gauthier and Jahsman, 1975).



A-A SECTION

Fig. 2. Experimental apparatus. A. Barrel of free-core LVDT. B. Bone specimen. C. Ball-bearing pulley. D. Dead weight. E. Wheel for mounting LVDT. F. Base plate. G. Plexiglass cell. H. Temperature probe. I. Thermoelectric heater. J. Adapter for loading.

INSTRUMENTATION AND PROCEDURE

Specimens of compact bone were obtained from human long bones from donated fresh-frozen autopsy tissue. Specimens were cut slowly, while wet, on a precision lathe into a cylindrical shape with flared, threaded ends as shown in Fig. 1, and were cut so that the long axis of the specimen was parallel to the direction of the osteons. Specimens 2–4 were from the femur of a male 175 cm (5' 9") in height, 59.1 kg in mass (130 lb weight) who died at age 27 of hepatic failure. Specimens 5 and 6 were from the right femur of a male 178 cm (5' 10") in height, 72.7 kg in mass (160 lb weight) who died at age 57 of Hodgkin's disease. These specimens are the same as those used by Yang and Lakes (1981) in a series of torsion tests to explore couple stress elastic behavior. Bone 1 was used only for a preliminary torsion experiment and is not included here.

During the course of experiments the specimens were kept in Ringer's solution with a bactericidal additive, and maintained at body temperature (36.5°C ± 0.2°C) by means of a closed loop temperature controller of original design. One end of the specimen was fixed on the base plate as shown in Fig. 2. The other end was connected with an adapter which enabled a step couple to be applied by means of an arrangement of dead weights, low friction ball bearings, and pulleys. A loading history approximated by a step function of time with a risetime of less than one second was applied to the specimen and sustained for periods of time greater than 240 s to constitute a bending creep experiment. In order to isolate the instrument from external vibration and shock, the base plate was mounted on a specially designed foundation composed of layers of sand, steel plate, cloth, a heavy steel block, and rubber. To reduce the error due to friction of the loading system, precision ball bearings used in the pulleys were vibrated in 'dither' at 3–4 kHz by magnetic coils or by piezoelectric transducers. The magnetic vibrator was more effective.

The radius of bending curvature R was determined by two DC to DC linear variable differential transformers (LVDTs), which were mounted on an adapter and were powered by a Tektronix power module. The LVDTs were calibrated using a bench micrometer. The two LVDT outputs were connected to a two channel Gould chart recorder. It was necessary to eliminate the effect of parasitic torsion of the specimen since the top torque rotor was unconstrained to minimize friction. This effect was minimized by adjusting the loading system until the difference of the LVDT outputs approached zero. The LVDT outputs were summed to eliminate any residual contribution to the results. Maximum strain did not exceed 10^{-4}, which ensured both material and geometrical linear behavior. Linearity of response was checked by repeating the tests at different load levels. Sufficient time for creep recovery was allowed between any two tests. After this, each specimen was cut to a smaller size, allowed to recover strains introduced in machining and the tests were repeated.

Analysis based on approximate couple stress theory was performed by assuming the equality to hold in equation (5) and defining $y \equiv J/d^2$, $x \equiv d^2$. A least-squares analysis was performed to obtain the coefficients B and A in $y = Bx + A$. Analysis based on micropolar theory (equation 8) was more complicated since the predicted size-dependence of the bending rigidity depends upon β, γ, κ, ν, and was carried out using an optimizing computer routine, VEO1AD, from the Harwell Subroutine Library. This subroutine minimizes a function of several variables subject to inequality constraints.

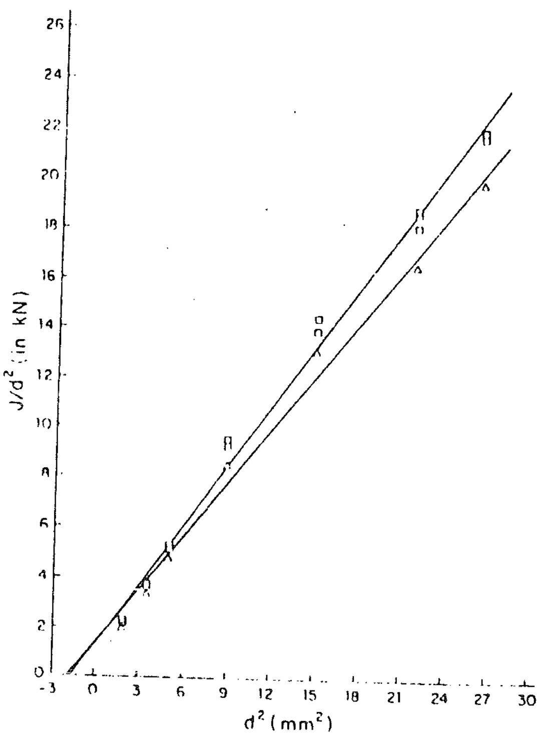


Fig. 3. Behavior of bone No. 4 in bending, curve fitting based on upper bound solution in couple stress theory. Squares: Isochronal experimental data for $t = 0.3$ s; triangles, $t = 240$ s. Solid lines are based on least squares analysis using equation (5).

RESULTS

Size-effect data were obtained from five specimens of compact bone. Data are evaluated both in terms of an upper bound analysis of bending in couple stress theory and an exact solution in micropolar theory, as discussed in the section on theory. Figure 3 displays experimental data points from a typical specimen and a straight line based on Koiter's couple stress solution fitted to the data by means of a least squares analysis. Table 1 shows the calculated Young's modulus and characteristic length for all five specimens, and Table 2 shows the associated confidence intervals. Calculations were performed on isochronal data, i.e. data from samples of different size at the same time following application of the step load. This procedure decoupled viscoelastic phenomena from the hypothesized couple-stress elastic phenomena so that the latter could be examined alone. All specimens give rise to regression lines offset from the origin as predicted by couple stress theory. The mean characteristic length in bending ℓ_{b1} is 0.148 mm and exhibits no significant dependence on time after loading, in the range 0.3 to 240 s. In Table 2, observe the one-sided confidence intervals for the regression line not to pass through the origin, i.e. for the constitutive equation for bone not to be that of linear elasticity. This confidence level is better than 99% for specimens 2, 4, 5, 6, at $t = 0.3$ s.

The characteristic length ℓ_{b1} is related to the total time period of testing T by the following regression equation: ℓ_{b1} [mm] $= 0.186 - 0.00220\,T$ [days]. The correlation coefficient $r = -0.842$ and $r^2 = 0.708$. We attribute this association to dissolution of part of the bone mineral during prolonged tests, an effect which is more severe for slender specimens which have a larger surface/volume ratio than thick specimens. Loss of apparent stiffness of thin specimens due to mineral dissolution results in an underestimate of the characteristic length. The above regression equation suggests that if all the experiments could be done in zero days, the characteristic length would appear larger; perhaps 0.18 mm.

The actual data for all specimens suggest that a curved plot is more appropriate than a straight line fit. This indicates that the equality in equation (3) is not satisfied and that if couple stress theory describes bone, $\eta'/\eta \neq -\nu$. A curved plot is predicted by an exact solution of the bending problem based on micropolar elasticity theory, for $\beta/\gamma \neq -\nu$. Data, therefore, were also analyzed on the basis of this solution. Figure 4 shows experimental points and a curve based on the exact solution in micropolar theory (equation 8). The 'best fit' theoretical curve shown was obtained by means of an optimizing computer routine. This program varied E, N, β/γ, and δ^{-1} until the sum of squares of the deviations between experimental points and the theoretical curve were minimized. Poisson's ratio ν was assumed to be 0.3. Observe that $N^2/\delta^2 = [\gamma/2(2\mu + \kappa)]$ and define this quantity as ℓ_{b2}^2. We

Table 1

Specimen	Load time (s)	A (Newton)	B (MN/m^2)	E (GN/m^2)	ℓ_{b1} (mm)	Time of test (days)
Bone No. 2	0.3	638 ± 191	954 ± 15	19.4	0.098	38
	120	599 ± 192	872 ± 15	17.8	0.098	
	240	585 ± 190	863 ± 15	17.6	0.098	
Bone No. 3	0.3	496 ± 499	889 ± 40	18.1	0.15	13
	120	518 ± 432	804 ± 35	16.4	0.098	
	240	475 ± 429	793 ± 34	16.1	0.098	
Bone No. 4	0.3	1370 ± 292	799 ± 19	16.3	0.16	13
	120	1415 ± 311	724 ± 21	14.8	0.17	
	240	1390 ± 303	716 ± 21	14.6	0.17	
Bone No. 5	0.3	1655 ± 257	752 ± 19	15.3	0.18	15
	120	1765 ± 223	675 ± 16	13.8	0.20	
	240	1760 ± 215	666 ± 16	13.6	0.20	
Bone No. 6	0.3	1110 ± 218	790 ± 19	16.1	0.15	9
	120	1200 ± 211	701 ± 19	14.3	0.16	
	240	1210 ± 212	687 ± 19	14.0	0.165	

may regard ℓ_{b2} as a micropolar characteristic length in bending, since its square is the ratio of a micropolar elastic constant to a classical elastic constant. For this specimen, $\ell_{b2} = 0.63$ mm. Figure 5 displays several theoretical curves in the vicinity of the optimal curve. ℓ_{b2} is held constant and N^2, δ^{-1}, and E are varied simultaneously. The results in bending do not permit a very accurate simultaneous evaluation of N^2 and δ^{-1}. The values of E, ℓ_{b2}, and β/γ are reasonably well determined, as illustrated in Fig. 4. Optimized micropolar coefficients for the five specimens examined in this study are displayed in Table 3. Specimen 3 exhibited a relatively large scatter, hence a large residual error; it was not included in the calculation of mean values.

DISCUSSION

Human compact bone appears to behave in a manner which is at variance with the predictions of classical elasticity or viscoelasticity theory. Experimental data are fitted more accurately by an exact solution of the bending problem in micropolar theory, than by an upper bound solution in indeterminate couple stress theory. In the micropolar analysis Young's modulus is smaller and the characteristic length is larger than in the approximate couple stress analysis. This occurs since in the latter case a straight line is fitted to a curved plot. Young's modulus is proportional to the limit, as specimen diameter d becomes large, of the slope of the curve in

Table 2

Specimen	Confidence interval for line not passing origin	Points	Confidence interval 90% A	90% B	95% A	95% B
Bone No. 2	99.2%	10	638 ± 355	954 ± 27.9	638 ± 440	954 ± 35
	99%	10	599 ± 359	872 ± 27.9	599 ± 445	872 ± 35
	98.8%	10	585 ± 355	963 ± 27.9	585 ± 438	863 ± 35
Bone No. 3	69.5%	12	496 ± 904	889 ± 72.5	496 ± 1000	889 ± 89
	75%	12	518 ± 783	804 ± 63	518 ± 962	804 ± 78
	71%	12	475 ± 759	793 ± 62	475 ± 934	793 ± 76
Bone No. 4	99.9%	15	1370 ± 517	799 ± 34	1370 ± 631	799 ± 41
	99.9%	14	1415 ± 554	724 ± 37	1415 ± 678	724 ± 46
	99.9%	14	1390 ± 540	616 ± 37	1390 ± 660	716 ± 46
Bone No. 5	99.9%	9	1655 ± 487	752 ± 36	1655 ± 608	752 ± 45
	99.9%	9	1765 ± 423	675 ± 30	1765 ± 527	675 ± 38
	99.9%	9	1760 ± 407	666 ± 30	1760 ± 508	666 ± 38
Bone No. 6	99.9%	10	1110 ± 405	790 ± 35	1110 ± 503	790 ± 44
	99.9%	10	1200 ± 392	701 ± 35	1200 ± 487	701 ± 44
	99.9%	10	1210 ± 394	687 ± 35	1210 ± 489	687 ± 44

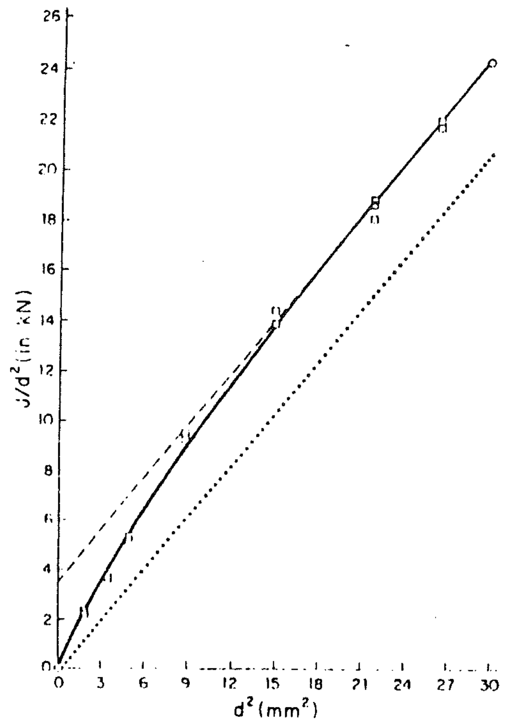


Fig. 4. Behavior of bone No. 4 in bending, curve fitting based on exact solution in micropolar theory. Solid curve is an optimal theoretical curve (equation 8) for $E = 12.367\ \mathrm{GN/m^2}$, $N = 1.0$, $\beta/\gamma = 1.0$, and $\delta^{-1} = 0.638$ mm. Squares represent experimental data. Dotted line represents the predicted behavior of a classically elastic material. Dashed line represents the predicted behavior of a micropolar solid for which $\beta/\gamma = -\nu$.

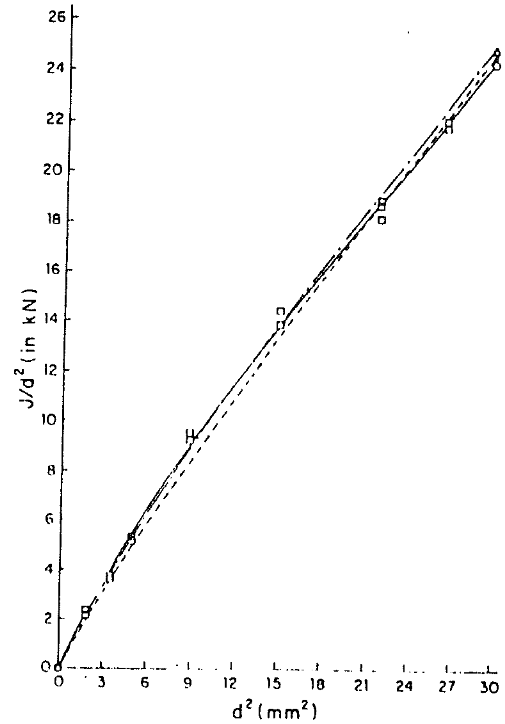


Fig. 5. Behavior of bone No. 4 in bending, curve fitting based on micropolar theory. Squares represent experimental data. Line ending in a circle represents the best fit theoretical curve for $E = 12.367\ \mathrm{GN/m^2}$, $N^2 = 1.0$, $\beta/\gamma = 1.0$ and $\delta^{-1} = 0.638$ mm. Solid line ending in a triangle represents a near optimal theoretical curve for $E = 12.900\ \mathrm{GN/m^2}$, $N^2 = 0.64$, $\beta/\gamma = 1.0$, and $\delta^{-1} = 0.798$ mm. Dashed line ending in a cross is for $E = 13.200\ \mathrm{GN/m^2}$, $N^2 = 0.32$, $\delta^{-1} = 1.063$ mm.

Fig. 3 or Fig. 4. The regression line in Fig. 3 overestimates this slope.

The values of Young's modulus obtained in this study may be compared with the results of Reilly, Burstein, and Frankel (1974), who measured Young's modulus E of human compact bone in tension at a strain rate 0.22 0.05 $\mathrm{s^{-1}}$. At this strain rate, the time to reach a strain 0.002–0.005 is ~ 0.1 s. They obtained the mean ± the standard deviation, $E = 17.1 \pm 3.15\ \mathrm{GN/m^2}$, presumably at room temperature. Smith and Walmsley (1952) observed that the bending stiffness of human bone decreases 0.25% per °F increase in temperature. Extrapolating from *ca.* 20 C to body temperature (37°C) we get $E = 15.7 \pm 2.9\ \mathrm{GN/m^2}$ in tension. The present couple stress analysis for $t \approx 0.3$ s gives $E = 17.0 \pm 1.67\ \mathrm{GN/m^2}$ and the micropolar analysis gives $E = 14.4 \pm 3.26\ \mathrm{GN/m^2}$, for human bone at 37°C. The micropolar Young's modulus is defined as

$$E = (2\mu + \kappa)(3\lambda + 2\mu + \kappa)(2\lambda + 2\mu + \kappa)^{-1}$$

by Gauthier and Jahsman (1975). This is equivalent to the classical form if $\mu + \kappa/2$ is identified with the shear modulus. The above E is what is measured in simple tension, in which there are neither strain gradients nor microrotations. This E is what is measured in bending only if the specimen is many times thicker than the characteristic length. Slender specimens are stiffened by micropolar effects; the maximum stiffening observed in this study is about a factor of two.

Isotropic micropolar and couple stress theory have been used in the present analysis of bone, which is anisotropic in its elastic properties. Although anisotropic micropolar constitutive equations are available (Eringen, 1968), the bending problem has been solved only in the isotropic case. Application of the isotropic solution to bone in the present study is successful since the material axes are aligned with the specimen symmetry axes. Isotropic theory cannot, however, be expected to adequately model the relationship between characteristic lengths measured in bending and in torsion, nor can it be used to extract the full set of micropolar elastic constants from experimental data for loading in different directions. Interpretation of the present results in the light of a bending solution based on anisotropic theory is a subject for future research.

Characteristic lengths obtained in this study are comparable to the size of osteons (*ca.* 0.25 mm diameter). Analytical models of structured materials described by continuum theories yield characteristic lengths comparable to the size of structural elements. We interpret the above to signify a dominant role for the osteon as a structural element of bone. Pickarski (1970) has examined the role of the osteon in the fracture of bone: 'pullout' of osteons can contribute

Table 3

Specimen	E (GN/m^2)	N^2	δ^{-1} (mm)	β/γ	ℓ_{b2} (mm)	Residual
Bone No. 2	19.1	0.50	0.232	1.0	0.16	0.75
Bone No. 3	12.0	0.75	0.971	1.0	0.84	7.37
Bone No. 4	12.4	1.00	0.629	1.0	0.63	1.63
Bone No. 5	12.0	1.00	0.605	1.0	0.61	0.22
Bone No. 6	14.1	0.65	0.489	1.0	0.39	0.56
Mean 2, 4, 5, 6	14.40	0.79	0.49	1.0	0.45	—
Standard deviation	3.26	0.25	0.18	0	0.22	—

substantially to the energy absorption to failure of bone. Katz (1980), in modelling the classical elasticity of bone on the basis of composite theory, considered the osteon to represent one level in a hierarchical analysis. Lakes and Saha (1979) examined the boundary between osteons as a mechanism for time dependent response of bone. The present study suggests a role for the osteon as a cause for nonclassical elastic behavior. The osteonal structure of human compact bone, then, has many consequences in the biomechanics of bone.

Ellis and Smith (1967) have commented negatively regarding the relationship between the characteristic length and the size of structural elements in metals: "In fact, should there be any couple stress effect at all, it would have to be associated with a value of ℓ ... of the order of the grain size .... In order for the couple stress to significantly alter the classical result one must deal with sheets which are 1 grain thick. Under such conditions, isotropic continuum theory will break down ..." We do not consider this view to be generally applicable to all materials which might be modelled by a generalized continuum theory. In the case of bone, there is a considerable range of specimen size for which micropolar theory rather than classical elasticity is clearly preferable. In this range we do not see that chaotic behavior which would characterize total breakdown of continuum theory. Eringen (1968) has anticipated this situation and has described such an intermediate zone of sizes as the region of the *micro-continuum*.

The potential significance of micropolar elasticity in bone is that stress fields predicted from this theory differ from the predictions of classical elasticity. The difference between these theories is most pronounced in regions of high strain gradient, e.g. near a hole or near an interface. Holes and interfaces are routinely generated in orthopaedic surgery on bone. Bone prosthesis systems are now analyzed on the basis of classical elasticity. Prostheses are designed in view of such stress analyses, to minimize the incidence of prosthesis loosening and failure, and bone resorption. We shall consider micropolar effects in the stress analysis of bone, in a future study, to ascertain whether they are important in clinically relevant situations.

CONCLUSION

Effects describable by the micropolar theory of elasticity have been observed in human compact bone. The average characteristic length for bending is 0.45 mm. Small specimens 2–3 mm in diameter are stiffened by about a factor of two by micropolar effects.

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NOMENCLATURE

a	Specimen radius, in mm
d	Specimen diameter, in mm
e_{ij}	Small strain tensor. $e_{ij} \equiv \frac{1}{2}(u_{i,j} + u_{j,i})$, dimensionless
e_{klm}	Permutation symbol
E	Young's modulus, in N/m^2
G	Shear modulus, in N/m^2. In micropolar elasticity, $G = \mu + (\kappa/2)$
J	Flexural rigidity, in $N \cdot m^2$
ℓ	Characteristic length as a constitutive parameter in couple stress theory, in mm
ℓ_{b1}	Characteristic length in bending in couple stress theory, in mm. $\ell_{b1} \equiv (G/E)^{1/2}\ell[1 + \nu^2 + 2\nu(\eta'/\eta)]^{1/2}$
ℓ_{b2}	Characteristic length in bending in micropolar theory, in mm. $\ell_{b2} \equiv N/\delta = [\gamma/(2(2\mu + \kappa))]^{1/2}$
ℓ_{b3}	Characteristic length in bending in micropolar theory, in mm. $\ell_{b3} = 2\ell_{b2}(G/E)^{1/2}$
m_{kl}	Couple stress tensor, in N/m
N	Coupling number in micropolar theory. $N^2 \equiv \kappa/2(\mu + \kappa)$, dimensionless
r_k	Macrorotation vector. $r_k \equiv e_{klm}u_{m,l}$, dimensionless
R	Bending radius of curvature, in mm
t_{kl}	Force stress tensor, in N/m^2
$\alpha, \beta, \gamma, \kappa$	Micropolar elastic constants in constitutive equation, α, β, γ in N; κ in N/m^2
δ^{-1}	Length parameter for bending of a micropolar rod. $\delta^{-1} \equiv [\gamma(\mu + \kappa)/\kappa(2\mu + \kappa)]^{1/2}$
δ_{ij}	Kronecker delta, dimensionless
η, η'	Couple stress elastic constants, in Newtons
ν	Poisson's ratio, dimensionless
Ω	Ratio of micropolar to classical flexural rigidity, dimensionless.