

**Biomechanics**  
**Quiz II**

name \_\_\_\_\_

Show all work and **all logic**. If needed feel free to ask for clarification. Enjoy!

1  
2  
3  
G

**Given:** 1 psi = 6.895 kPa, 1 lb = 4.48 N, 1 inch = 25.4 mm.  $g = 9.81 \text{ m/sec}^2 = 32.2 \text{ ft/sec}^2$ .  
60 mph = 88 ft/sec. 1 m = 3.281 ft. 1 m/sec = 2.237 mi/hr.  $E = 2G(1 + \nu)$ .  $\mathbf{F} = m\mathbf{a}$ .  $\mathbf{M} = I\alpha$

$$\sigma_{xx} = My/I; \Omega = 1/\rho = M/EI. I = \frac{1}{12} bh^3. I = \frac{\pi}{4} r^4. I_c = \frac{\pi}{4} (r_o^4 - r_i^4). I_{el} = \frac{\pi}{4} (a_o b_o^3 - a_i b_i^3).$$

$$\epsilon_{xx} = \frac{1}{E} \{ \sigma_{xx} - \nu\sigma_{yy} - \nu\sigma_{zz} \}. \sigma_{yx} = Mr/J; J = \frac{\pi}{2} (r_o^4 - r_i^4). \gamma_{yz} = r\phi/L. ML/\phi = GJ.$$

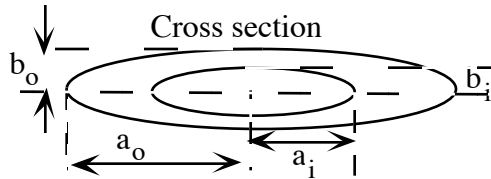
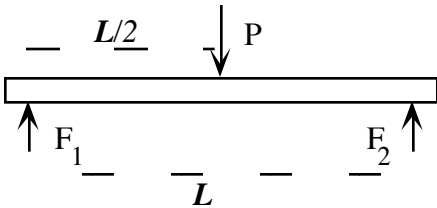
Material	Young's modulus (GPa)	Ultimate strength (MPa)	Yield strength (MPa)	density (g/cm <sup>3</sup> )
Structural steel	200	400, tension	250, tension	7.9
Compact bone	17	130, tension 220, compression	80, tension	2.0

**Given:**  $C_{1111} =$   
 $E \frac{1 - \nu}{(1 + \nu)(1 - 2\nu)}$   
 $\sigma = \frac{P}{A}$ .  $P = F \nu$   
 $v_s = \sqrt{G/\rho}$ .  
 $v_L = \sqrt{C_{1111}/\rho}$ .  
 $v_f^2 - v_i^2 = 2ay$ .  
 $P = dW/dt$ .

1. (30 pts) Consider the **strength to weight ratio** of animals or humans of different size (height)  $L$  but identical proportions and body composition.

(a) (25 pts) Determine the ratio of limb strength to weight as a function of  $L$ . Assume that the limb strength (maximum limb force) is limited by the maximum torsional stress in bone.

(b) (5 pts) Two people do pull-up exercises. One weighs 160 pounds; the other weighs 320 pounds. They have essentially the same proportions and body composition. Who can do more pull-ups? Why?



2. (30 pts) Consider three point bending. A transverse force  $P$  acts midway along the bar.

(a) Find how the bend moment  $M(x)$  varies with position  $x$  along the bar specimen, from  $x = 0$  to  $x = L/2$ .  $L$  is the length of the specimen between supports.  $P$  acts at  $x = L/2$ . Hint. Draw a free body diagram.

(b) Show that for a three point bend test, the force  $P$  at the center is given by  $P = 4 \sigma I / L y_{max}$ , with  $L$  as the length of the specimen between supports,  $I$  its moment of inertia,  $y_{max}$  the maximum distance from the neutral axis,  $\sigma$  the maximum stress at the surface.

(c) A compact bone specimen has  $L = 150 \text{ mm}$ , and an elliptic cross section with semi-major dimensions outer  $a_o = 20 \text{ mm}$ , and inner  $a_i = 15 \text{ mm}$ ; semi-minor dimensions outer  $b_o = 15 \text{ mm}$ , and inner  $b_i = 10 \text{ mm}$ , how much force is needed to break it?



*Australian giant rat has tendons.*

3. (40 pts) Consider stress-strain curves.

(a) Draw tensile and compressive stress strain curves for compact bone. Label both axes and show numerical values for yield strength and yield strain. Bone is essentially linear below the yield point.

(b) Sketch a tensile stress strain curve for tendon.

(c) Suppose a tissue has a tensile stress strain relation  $\sigma = 5500 \epsilon^2$ , with stress  $\sigma$  in MPa and strain  $\epsilon$  less than 0.06. What is the stress at a strain of 0.03?

(d) What is the force at a strain of 0.03 if the above tissue in (c) is cylindrical and 6 mm diameter?

(e) What is the biological advantage of a stress-strain relation such as that in (c) ?

What kind of histology (microscopic structure) can give rise to such a stress-strain curve?