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NONCENTROSYMMETRY IN MICROPOLAR ELASTICITY

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Abstract—Consequences of noncentrosymmetry in a micropolar elastic solid are considered. A solid which is isotropic with respect to coordinate rotations but not with respect to inversions, has three new elastic constants in addition to the six considered in the fully isotropic micropolar solid. The acentric micropolar solid is predicted to undergo torsional deformation when subjected to tensile load.

1. INTRODUCTION

GENERALIZED continuum theories for mechanical behavior developed over the last century admit degrees of freedom not considered in the classical theory of elasticity. Common to such theories as those of the Cosserats[1], the indeterminate couple stress theory of Mindlin and Tiersten[2] and the micropolar theory of Eringen[3] is the assumption of couple stress and the associated asymmetry of the force stress tensor. Generalized continuum theories are thought to have applications in the modelling of materials with microstructure, such as granular or fibrous materials, or materials with a lattice structure. Micropolar theory has in recent years stimulated considerable interest, and analytical solutions to many problems in micropolar elasticity are available. Of particular interest to the experimentalist are the predictions of size effects in the apparent stiffness of a cylindrical member in torsion[4] and in bending[5]. In most published solutions, material isotropy is assumed. Some materials, however, are not invariant to coordinate inversions and this type of anisotropy can be expected to result in qualitatively different behavior in comparison with isotropic solids. Some aspects of initial stress in noncentrosymmetric Cosserat continua have been examined[6] but geometries addressable experimentally were not considered. Structural noncentrosymmetry is characteristic of bone, as well as synthetic composites containing twisted fibers. In this paper the behavior of a noncentrosymmetric micropolar elastic solid is examined.

2. NONCENTROSYMMETRIC MICROPOLAR THEORY

In the linear theory of an anisotropic micropolar solid, the free energy Ψ is given in terms of the microrotation ϕ and the micropolar strain $\epsilon_{kl} \equiv e_{kl} + e_{klm}(r_m - \phi_m)$ by[3]

$$\rho\psi = A_0 + A_{kl}\epsilon_{kl} + \frac{1}{2}A_{klmn}\epsilon_{kl}\epsilon_{mn} + B_{kl}\phi_{k,l}$$

$$+\frac{1}{2}B_{klmn}\phi_{k,l}\phi_{m,n} + C_{klmn}\epsilon_{kl}\phi_{m,n}, \tag{2.1}$$

in which ρ is the density, and the A's, B's and C's are elastic constants. In the expression for micropolar strain, $e_{kl} \equiv 1/2(u_{k,l} + u_{l,k})$ is the usual macrostrain tensor, defined in terms of the displacement u, e_{klm} is the permutation symbol, and $r_k \equiv 1/2(e_{klm}u_{m,l})$ is the macrorotation vector. The usual Einstein summation convention is used, and the comma denotes partial differentiation with respect to spatial coordinates. The stress σ_{kl} and the couple stress m_{kl} are given by

$$\sigma_{kl} = \partial\rho\Psi/\partial\epsilon_{kl},\ m_{kl} = \partial\rho\Psi/\partial\phi_{l,k}. \tag{2.2}$$

In the absence of initial stress $A_{kl} = 0$ and in the absence of initial couple stress $B_{kl} = 0$. The constitutive equations, obtained from (2.2) and (2.1) are

$$\sigma_{kl} = A_{klmn}\epsilon_{mn} + C_{klmn}\phi_{m,n} \tag{2.3}$$

$$m_{kl} = B_{klmn}\phi_{m,n} + C_{mnlk}\epsilon_{mn}. \quad (2.4)$$

Now ϕ_k is an axial vector, therefore, the terms in eqn (2.1) containing C_{klmn} and B_{kl} change sign under an inversion of spatial axes. The other terms do not change sign, therefore, the internal energy is not invariant to such inversions if $C_{klmn} \neq 0$ or $B_{kl} \neq 0$. This lack of invariance is permitted if the material does not have a center of symmetry. The case of centrosymmetric, isotropic materials has been treated at great length in the literature. In the present analysis, we consider a material which is noncentrosymmetric but is isotropic with respect to coordinate rotations.

The most general fourth order isotropic tensor may be written

$$D_{klmn} = D_1\delta_{kl}\delta_{mn} + D_2\delta_{km}\delta_{ln} + D_3\delta_{kn}\delta_{lm}. \quad (2.5)$$

The constitutive eqns (2.3) and (2.4) become

$$\sigma_{kl} = A_1\epsilon_{rr}\delta_{kl} + A_2\epsilon_{kl} + A_3\epsilon_{lk} + C_1\phi_{r,r}\delta_{kl} + C_2\phi_{k,l} + C_3\phi_{l,k} \quad (2.6)$$

$$m_{kl} = B_1\phi_{r,r} + B_2\phi_{l,k} + B_3\phi_{k,l} + C_1\epsilon_{rr}\delta_{kl} + C_2\epsilon_{lk} + C_3\epsilon_{kl}. \quad (2.7)$$

In terms of the macrostrain e_{kl} and conventional notation for the elastic constants, these may be written

$$\sigma_{kl} = \lambda e_{rr}\delta_{kl} + (2\mu + \kappa)e_{kl} + \kappa e_{klm}(r_m - \phi_m) + C_1\phi_{r,r}\delta_{kl} + C_2\phi_{k,l} + C_3\phi_{l,k} \quad (2.8)$$

$$m_{kl} = \alpha\phi_{r,r}\delta_{kl} + \gamma\phi_{l,k} + \beta\phi_{k,l} + C_1 e_{rr}\delta_{kl} + (C_2 + C_3)e_{kl} + (C_3 - C_2)e_{klm}(r_m - \phi_m) \quad (2.9)$$

These are the constitutive equations for a micropolar solid which is isotropic with respect to coordinate rotations but not with respect to inversions. Elastic constants C_1, C_2 and C_3 are associated with noncentrosymmetry; if these vanish, the equations of isotropic micropolar elasticity are recovered. The quantities α, β, γ and κ are micropolar elastic constants; if these also vanish, eqns (2.8) and (2.9) reduce to the constitutive equations of classical isotropic, linear elasticity theory, in which λ and μ are the Lamé constants. Boundary conditions do not depend on assumed material symmetry. One may prescribe the displacements u_k or the surface traction $t_{(n)k}$ and the microrotations ϕ_k or the surface couples $m_{(n)k}$ on the surface which has exterior normal n_l. If tractions are specified, the boundary conditions are[3]

$$\sigma_{lk}n_l = t_{(n)k} \quad (2.10)$$

$$M_{lk}n_l = m_{(n)k}. \quad (2.11)$$

The laws of motion also are independent of material symmetry and are given by[3]

$$\sigma_{ik,l} + \rho(f_k - a_k) = 0 \quad (2.12)$$

$$m_{lk,l} + e_{kmn}\sigma_{mn} + \rho(\lambda_k - \alpha_k) = 0, \quad (2.13)$$

In which f_k is the body force, a_k is the local acceleration, λ_k is the body couple and α_k is the local angular acceleration.

3. RESTRICTIONS ON MICROPOLAR ELASTIC MODULI

In order that our noncentrosymmetric micropolar solid be stable, it is necessary that the internal energy be nonnegative. From this requirement, one may obtain restrictions on the micropolar elastic moduli. Consider the internal energy:

$$\begin{aligned}\rho\Psi = {} & 1/2[\lambda e_{kk}e_{ll} + (2\mu + \kappa)\, e_{kl}e_{kl}] + [\kappa(r_k - \phi_k)(r_k - \phi_k)] \\ & + 1/2[\alpha\phi_{k,k}\phi_{l,l} + \beta\phi_{k,l}\phi_{l,k} + \gamma\phi_{k,l}\phi_{k,l}] \\ & + 1/2[C_1 e_{kk}\phi_{l,l} + C_2 e_{kl}\phi_{l,k} + C_2 e_{klm}(r_m - \phi_m)\phi_{l,k} + C_3 e_{kl}\phi_{k,l} \\ & C_3 e_{klm}(r_m - \phi_m)\phi_{k,l}]. \end{aligned} \tag{3.1}$$

Using the definitions $\phi_{(k,l)} \equiv (\phi_{k,l} + \phi_{l,k})/2$, $\phi_{[k,l]} \equiv (\phi_{k,l} - \phi_{l,k})/2$ we may rewrite the energy

$$\begin{aligned}2\rho\Psi = {} & [\lambda e_{kk}e_{ll} + (2\mu + \kappa)e_{kl}e_{kl}] + [2\kappa(r_k - \phi_k)(r_k - \phi_k)] \\ & + [1/3(3\alpha + \beta + \gamma)\phi_{k,k}\phi_{l,l}] + [(\gamma - \beta)\phi_{[k,l]}\phi_{[k,l]}] \\ & + (\gamma + \beta)[\phi_{(k,l)} - 1/3\phi_{r,r}\delta_{kl}][\phi_{(k,l)} - 1/3\phi_{s,s}\delta_{kl}] \\ & + [1/3 e_{kk}\phi_{l,l}(3C_1 + C_2 + C_3)] + [\phi_{[l,k]}e_{kl}(C_2 - C_3)] \\ & + [(\phi_{(l,k)} - 1/3\phi_{s,s}\delta_{lk})e_{lk}(C_2 + C_3)] \\ & + [(r_m - \phi_m)(C_2 e_{klm}\phi_{l,k} + C_3 e_{klm}\phi_{k,l})]. \end{aligned} \tag{3.2}$$

Observe that the quantities e_{kl}, $\phi_{[l,k]}$ and $(r_k - \phi_k)$ can be varied independently of one another. If the first bracketed term in eqn (3.2) is the only one present, the requirement that this term be nonnegative yields

$$3\lambda + 2\mu + \kappa \geqslant 0,\, 2\mu + \kappa \geqslant 0 \tag{3.3}$$

as in classical elasticity; the quantity $\mu + \kappa/2$ is identified with the Lamé shear modulus. The energies represented by the second, third and fourth terms must each be nonnegative, so for Ψ to be nonnegative it is necessary that

$$\kappa \geqslant 0,\, 3\alpha + \beta + \gamma \geqslant 0,\, -\gamma \leqslant \beta \leqslant \gamma, \tag{3.4}$$

a result obtained by Eringen for the fully isotropic micropolar solid[3].

The product terms containing both e and ϕ cannot exist independently of the first—fourth terms, therefore, the above approach cannot be used to restrice the C coefficients; they can be positive or negative. However, for Ψ to be nonnegative, it is necessary that a negative product term not be greater in magnitude than the sum of the corresponding positive terms containing e and ϕ individually. For example if $\phi_{k,l}$ and e_{kl} are traceless, $\phi_{[k,l]} = 0$ and $r_k = \phi_k$, it is necessary that

$$K_{10}^2 \equiv \frac{(C_2 + C_3)^2}{4(2\mu + \kappa)(\beta + \gamma)} \leqslant 1 \tag{3.5}$$

for the energy Ψ to be nonnegative. Similarly, considering other possible combinations, one obtains

$$K_{11}^2 = \frac{(C_2 - C_3)^2}{4(2\mu + \kappa)(\gamma - \beta)} \leqslant 1 \tag{3.6}$$

$$K_{12}^2 = \frac{(3C_1 + C_2 + C_3)^2}{4(3\lambda + 2\mu + \kappa)(3\alpha + \beta + \gamma)} \leqslant 1 \tag{3.7}$$

as necessary conditions for Ψ to be nonnegative. The C coefficients associated with noncentrosymmetry are bounded by products of combinations of classical elastic and micropolar coefficients. The quantities K_{10}^2, K_{11}^2, K_{12}^2 are analogous to the coupling coefficients developed in the linear theory of piezoelectricity, and can be obtained in a similar fashion. This correspondence is anticipated on the basis of the formal similarity between the constitutive equations of linear piezoelectricity and those of noncentrosymmetric micropolar elasticity, eqns (2.3) and (2.4).

4. SIMPLE TENSION

Consider a cylindrical rod of radius R, of a noncentrosymmetric micropolar elastic solid. Let the rod be stretched by an axial force F, and let it be free of rotational constraint, and let the lateral surface be free of force traction and couples. Such a situation is relatively easy to realize experimentally, and serves to illustrate the effects of the C coefficients. To solve this tension problem, it is useful to express the constitutive equations, equilibrium equations and strain-displacement relations in cylindrical polar coordinates. The constitutive equations may be written

$$\begin{bmatrix} t_{rr} \\ t_{\theta\theta} \\ t_{zz} \end{bmatrix} = \lambda\epsilon_{jj} + (2\mu+\kappa)\begin{bmatrix} \epsilon_{rr} \\ \epsilon_{\theta\theta} \\ \epsilon_{zz} \end{bmatrix} + C_1\frac{1}{r}(r\phi_r)_{,r} + \frac{1}{r}\phi_{\theta,\theta} + \phi_{z,z} + (C_2+C_3)\begin{bmatrix} \phi_{r,r} \\ \phi_r/r \\ \phi_{z,z} \end{bmatrix} + \phi_{\theta,\theta}/r$$

$$\begin{bmatrix} t_{r\theta} \\ t_{rz} \\ t_{\theta r} \\ t_{\theta z} \\ t_{zr} \\ t_{z\theta} \end{bmatrix} = (\mu+\kappa)\begin{bmatrix} \epsilon_{r\theta} \\ \epsilon_{rz} \\ \epsilon_{\theta r} \\ \epsilon_{\theta z} \\ \epsilon_{zr} \\ \epsilon_{z\theta} \end{bmatrix} + \mu\begin{bmatrix} \epsilon_{\theta r} \\ \epsilon_{zr} \\ \epsilon_{r\theta} \\ \epsilon_{z\theta} \\ \epsilon_{rz} \\ \epsilon_{\theta z} \end{bmatrix} + C_2\begin{bmatrix} -\phi_\theta/r \\ \phi_{r,z} \\ \phi_{\theta,r} \\ \phi_{\theta,z} \\ \phi_{z,r} \\ \phi_{z,\theta} \end{bmatrix} + C_3\begin{bmatrix} \phi_{\theta,r} \\ \phi_{z,r} \\ -\phi_\theta/r \\ \phi_{z,\theta} \\ \phi_{r,z} \\ \phi_{\theta,z} \end{bmatrix}$$

$$\begin{bmatrix} m_{rr} \\ m_{\theta\theta} \\ m_{zz} \end{bmatrix} = \alpha\frac{1}{r}(r\phi_r)_{,r} + \frac{1}{r}\phi_{\theta,\theta} + \phi_{z,z} + (\beta+\gamma)\begin{bmatrix} \phi_{r,r} \\ \phi_r/r \\ \phi_{z,z} \end{bmatrix} + C_1\epsilon_{ii} + (C_2+C_3)\begin{bmatrix} \epsilon_{rr} \\ \epsilon_{\theta\theta} \\ \epsilon_{zz} \end{bmatrix}$$

$$\begin{bmatrix} m_{r\theta} \\ m_{rz} \\ m_{\theta r} \\ m_{\theta z} \\ m_{zr} \\ m_{z\theta} \end{bmatrix} = \beta\begin{bmatrix} \phi_\theta/r \\ \phi_{r,z} \\ \phi_{\theta,r} \\ \phi_{\theta,z} \\ \phi_{z,r} \\ \phi_{z,\theta} \end{bmatrix} + \gamma\begin{bmatrix} \phi_{\theta,r} \\ \phi_{z,r} \\ \phi_\theta/r \\ \phi_{z,\theta} \\ \phi_{r,z} \\ \phi_{\theta,z} \end{bmatrix} + C_2\begin{bmatrix} \epsilon_{r\theta} \\ \epsilon_{rz} \\ \epsilon_{\theta r} \\ \epsilon_{\theta z} \\ \epsilon_{zr} \\ \epsilon_{z\theta} \end{bmatrix} + C_3\begin{bmatrix} \epsilon_{\theta r} \\ \epsilon_{zr} \\ \epsilon_{r\theta} \\ \epsilon_{z\theta} \\ \epsilon_{rz} \\ \epsilon_{\theta z} \end{bmatrix}.$$

The equilibrium equations are

$$\begin{bmatrix} t_{rr,r} \\ t_{r\theta,r} \\ t_{rz,r} \end{bmatrix} + \frac{1}{r}\begin{bmatrix} t_{rr} - t_{\theta\theta} \\ t_{r\theta} + t_{\theta r} \\ t_{rz} \end{bmatrix} = 0$$

$$\begin{bmatrix} m_{rr,r} \\ m_{r\theta,r} \\ m_{rz,r} \end{bmatrix} + \frac{1}{r}\begin{bmatrix} m_{rr} - m_{\theta\theta} \\ m_{r\theta} + m_{\theta r} \\ m_{rz} \end{bmatrix} + \begin{bmatrix} t_{\theta z} - t_{z\theta} \\ t_{zr} - t_{rz} \\ t_{r\theta} - t_{\theta r} \end{bmatrix} = 0.$$

The micropolar strains in terms of the displacements u and microrotations ϕ are

$$\begin{aligned} &\epsilon_{rr} = u_{r,r} && \epsilon_{r\theta} = u_{\theta,r} - \phi_z && \epsilon_{\theta r} = -u_\theta/r + \phi_z \\ &\epsilon_{\theta\theta} = u_r/r && \epsilon_{rz} = u_{z,r} + \phi_\theta && \epsilon_{zr} = u_{r,z} - \phi_\theta \\ &\epsilon_{zz} = u_{z,z} && \epsilon_{\theta z} = -\phi_r && \epsilon_{z\theta} = u_{\theta,z} + \phi_r \end{aligned}$$

In the case of simple tension of a long cylindrical rod of radius R, the following field of displacement and microrotation gives rise to a solution

$$\left\{\begin{array}{ll} u_z = ez & \phi_z = b_0 z \\ u_r = -\left(\nu_0 r + \dfrac{A_9(C_1+C_2+C_3)}{e(\lambda+2\mu+\kappa)} I_1(pr)\right)e & \phi_r = A_9 I_1(pr) - \dfrac{b_0}{2} r \\ u_\theta = b_0 rz & \phi_\theta = 0, \end{array}\right\}$$

in which

$$p^2 = \frac{2\kappa}{\alpha+\beta+\gamma}\frac{1}{1-K_0^2}, \quad K_0^2 = \frac{(C_1+C_2+C_3)^2}{(\alpha+\beta+\gamma)(\lambda+2\mu+\kappa)},$$

and I_1 is the modified Bessel function of first order.

We may identify e with the axial strain and B_0 with a twist angle per unit length of rod, arising from the coupling produced by the C coefficients. We observe that the Poisson-like contraction is not associated with a uniform radial strain as is the case in classical elasticity or in centrosymmetric micropolar elasticity [4].

The quantities b_0, ν_0 and A_9 are obtained by solving simultaneously the boundary condition equations for zero force traction and zero couple on the lateral surface of the rod, and zero net torque at the ends. The values are

$$b_0 = \frac{\begin{vmatrix} -l_1R^2e/2 & -l_2R^2e & l_3^2A_{11} - K_1^2(l_2^2+l_3^2)A_{11} + K_5A_{12} \\ -K_3K_0e/l_4 & -K_0(K_3+1)e/l_4 & A_{30}(1-K_0^2) + (K_2+K_1^2)I_1(pR)/R \\ -K_4e/l_4K_0 & -(K_4+1)e/K_0l_4 & (K_3-K_4)I_1(pR)/R \end{vmatrix}}{D}$$

$$\nu_0 = \frac{\begin{vmatrix} (l_0^2+(R/2)^2)R^2/2 & -l_2R^2e/2 & l_3^2A_{11} - K_1^2(l_2^2+l_3^2)A_{11} + K_5A_{12} \\ (K_2-1)/2 & -K_3K_0e/l_4 & A_{30}(1-K_0^2) + (K_2-K_1^2)I_1(pR)/R \\ (K_3-1)/2 & -K_4e/K_0l_4 & (K_3-K_4)I_1(pR)/R \end{vmatrix}}{D}$$

$$A_9 = \frac{\begin{vmatrix} (l_0^2+(R/2)^2)R^2/2 & -l_2R^2e & -l_1R^2e/2 \\ (K_2-1)/2 & -K_0(K_3+1)e/l_4 & -K_3K_0e/l_4 \\ (K_3-1)/2 & -(K_4+1)e/K_0l_4 & -K_4e/K_0l_4 \end{vmatrix}}{D}$$

in which

$$D = \begin{vmatrix} (l_0^2+(R/2)^2)R^2/2 & -l_2R^2e & l_3^2A_{11} - K_1^2(l_2^2+l_3^2)A_{11} + K_5A_{12} \\ (K_2-1)/2 & -(K_3+1)K_0e/l_4 & A_{30}(1-K_0^2) + (K^2-K_1^2)I_1(pR)/R \\ (K_3-1)/2 & -(K_4+1)e/K_0l_4 & (K_3-K_4)I_1(pR)/R. \end{vmatrix}$$

In the above, the following quantities have been defined in terms of the micropolar elastic constants

$$l_0^2 = (\beta+\gamma)/(2\mu+\kappa),\ l_4^2 = (\alpha+\beta+\gamma)/(\lambda+2\mu+\kappa),$$

$$K_0^2 = (C_1+C_2+C_3)^2/(\alpha+\beta+\gamma)(\lambda+2\mu+\kappa)$$

$$K_2 = \alpha/(\alpha+\beta+\gamma),\ K_3 = C_1/(C_1+C_2+C_3),$$

$$K_4 = \lambda/(\lambda+2\mu+\kappa),\ K_5 = \kappa/(2\mu+\kappa).$$

Additional quantities are defined as

$$l_1 = l_4 K_0/(1 - K_4),\ l_2 = l_1 K_3,\ l_3^2 = (l_4^2/(1 - K_4)) - l_0^2,$$
$$K_1^2 = K_3 K_0^2,\ A_{30} = (pI_0(pR) - I_1(pR)/R),$$
$$A_{11} = RI_1(pR),\ A_{12} = \int_0^R r^2 I_1(pR)\,\mathrm{d}r = R^2 I_2(pR)/p,$$

in which I_0, I_1 and I_2 are modified Bessel functions of order zero, one and two, respectively.

It is instructive to examine several special cases. If $C_1 = 0$, $\lambda = 0$ and $\alpha = 0$, the twist angle per unit length is

$$b_0 = \frac{-K_0 l_0}{\left(l_0^2 + \left(\frac{R}{2}\right)^2\right) + \frac{R^2 I_2(pR)}{(pR)^2 I_0(pR) - pRI_1(pR)} K_5}\, e.$$

The twist angle is proportional to the axial strain e and to the coupling factor K_0, and it tends to increase as the cylinder radius decreases. The twist angle can be positive or negative, depending on the sign of K_0. A second special case is obtained by constraining the microrotation to be equal to the macrorotation. This constraint yields a solution to the tension problem in noncentrosymmetric indeterminate couple stress theory. The constraint is achieved by allowing p to become infinitely large. The twist angle per unit length becomes

$$b_0 = \frac{-K_0 l_4[1 - 2K_3K_4/(1 + K_4) + [(K_3 + 1)K_4/(1 + K_4) - K_3)]/(1 - K_4)}{\left(\frac{R}{2}\right)^2 + l_0^2 + l_4^2[K_0K_3(K_3 - 1)/(1 - K_4^2) + ((1 - K_2) - K_0^2(1 - K_3^2)/(1 + K_4))/2(1 - K_4)]}\, e.$$

In both special cases the twist angle is proportional to the coupling factor K_0 and the axial strain e. For a thick rod of radius $R \gg l_0$, the twist angle increases as the inverse square of the radius. For a sufficiently small radius, $R \ll l_0$, the twist angle per unit axial strain approaches a constant value.

5. PHYSICAL INTERPRETATION

The quantities $l_0 - l_4$ have dimensions of length and may be referred to as characteristic lengths. l_0 is the characteristic length defined in connection with the problem of torsion in centrosymmetric micropolar theory by Gauthier and Jahsman[4]. In generalized continuum models of structured materials, the characteristic lengths are generally found to be related to the size of structural elements. The quantities $K_0 - K_5$ are dimensionless. K_0 and K_1 are measures of the strength of the noncentrosymmetric coupling and are analogous to the coupling coefficients of piezoelectricity theory. K_4 is equivalent to the classical Poisson ratio, since $\mu + \kappa/2$ is the observed shear modulus. K_2 and K_3 are similar in nature to K_4, as seen by comparing the role of λ with that of C_1 and α in eqns (2.8) and (2.9). K_5 represents the strength of coupling between the macrostrain field and the microrotation field.

Micropolar elasticity and related continuum theories are thought to apply to granular, fibrous, or composite materials. The noncentrosymmetric theory is intended for solids containing twisted or spiraling fibers, in which one direction of twist or spiral predominates. If the fibers are distributed randomly in all directions, tensile specimens taken in any orientation will appear to have the same Young's modulus, giving the impression of isotropy.

6. EXPERIMENTAL

Few experiments of any kind have been performed to explore micropolar effects in real materials. Efforts to find effects describable by indeterminate couple stress theory in metals and by micropolar theory in a composite have been unsuccessful. Recently one of the authors (R.L.) has found evidence of couple stress effects in human compact bone[7, 8]. There is some indication that micropolar theory is to be preferred over indeterminate couple stress theory in describing these effects. Regarding effects due to noncentrosymmetry, positive but very preliminary experimental results have been found[9].

Ropes and cables containing fibers which spiral are structurally noncentrosymmetric. It is well known that they untwist when subjected to tensile force with no constraint on rotation, as predicted in Section 4. Use of a continuum model for ropes is, however, questionable.

Future experiments seeking to demonstrate micropolar behavior could be performed using the tension mode described in Section 4. This is an attractive modality since great sensitivity is possible. It should be possible to detect acentric micropolar effects even if the structural asymmetry is on the atomic or molecular scale. For example, a fiber 0.07 mm dia. and 200 mm long is typical of boron-epoxy fibers used in composites. If such a fiber were subjected to an axial strain of 10^{-3} and if $l_0 = 10\,\text{Å} = 10^{-9}\,\text{m}$ and $K_0 = 1$, the untwisting due to noncentrosymmetry could be detected. A reflected laser beam over an 8 m path would be deflected about 2 mm by rotation of the fiber.

Experiments based on the results in Section 4 are capable of detecting micropolar behavior, but calculation of the nine elastic constants will be less than straightforward. A similar complexity in the combination of elastic constants is found in earlier work on centrosymmetric micropolar theory[4, 5]. From the experimentalist's point of view, it appears that further attention to the solution of micropolar boundary value problems is warranted.

7. DISCUSSION

Several consequences of noncentrosymmetry in micropolar elasticity have been considered. Torsion deformation in response to tensile load, and size effects in Poisson's ratio are predicted. Very sensitive experiments based on the predicted behavior are possible. Macroscopic noncentrosymmetric micropolar effects may occur in composite materials with twisted or spiraling fibers.

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REFERENCES

[1] E. and F. COSSERAT, *Theorie des Corps Deformables*. Hermann et Fils, Paris (1909).
[2] R. D. MINDLIN and H. F. TIERSTEN, *Arch. Rat. Mech. Anal.* **11**, 415 (1962).
[3] A. C. ERINGEN, In *Fracture* (Edited by R. Liebowitz), **2**, 621, 729 (1968).
[4] R. D. GAUTHIER and W. E. JAHSMAN, *J. Appl. Mech.* **42**, 369, 374 (1975).
[5] G. V. KRISHNA REDDY and N. K. VENKATASUBRAMANIAN, *J. Appl. Mech.* **45**, 429 (1978).
[6] Y. WEITSMAN, *J. Appl. Mech* **34**, 160 (1967).
[7] R. S. LAKES, *J. Biomechanical Engng* **104**, 6, 11 (1982).
[8] J. F. C. YANG and R. S. LAKES, *J. Biomechanical Engng* **103**, 275, 279 (1981).
[9] R. S. LAKES, *Proc.* 34th *ACEMB*. Houston (1981).

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