VISCOELASTIC SOLIDS
Quiz 2

Show **logic** and state all principles and assumptions used.

Given: $J''(\omega) = \frac{2\omega}{\pi}\left[\wp \int_0^\infty \frac{J'(\varpi)-J'(\infty)}{\varpi^2-\omega^2} d\varpi \right]$ $\quad J'(\omega) - J'(\infty) = \frac{2}{\pi}\left[\wp \int_0^\infty \frac{\varpi J''(\varpi)}{\varpi^2-\omega^2} d\varpi \right]$, $\sigma(t) = \int_0^t E(t-\tau)\frac{d\varepsilon(\tau)}{d\tau} d\tau$; 1 N = 10^5 dyne

$\mathcal{L}[f(t)] \equiv F(s) \equiv \int_0^\infty f(t)e^{-st}dt$, $\nu = \frac{3B-2G}{6B+2G} = \frac{1}{2} - \frac{E}{6B}$, $E = 2G[1+\nu]$, $x(t) \approx x_0\, e^{-(\omega t/2)\tan\delta} \sin \omega t$. $\mathcal{L}[\frac{df(t)}{dt}] = s\mathcal{L}[f(t)] - f(0)$, $E = 2G(1+\nu)$

$\int \frac{x}{(x^2-a^2)^2}dx = -\frac{1}{2(x^2-a^2)}$. $\text{ctn } x \cong \text{ctn }[\pi/2] + [-\csc^2(\pi/2)][x-\pi/2] + \ldots$ $\quad \mathcal{L}[e^{-at}] = \frac{1}{s+a}$, $\mathcal{L}[1] = \frac{1}{s}$, $\mathcal{L}[\mathcal{H}(t)] = \frac{1}{s}$,

$\mathcal{L}[t] = \frac{1}{s^2}$, $\mathcal{L}[\mathcal{H}(t-a)] = e^{-as}/s$, $\mathcal{L}[\delta(t-a)] = e^{-as}$, $\mathcal{L}[t^n e^{-at}] = \frac{n!}{(s+a)^{n+1}}$, $\mathcal{L}[\frac{t^{n-1}e^{at}}{(n-1)!}] = \frac{1}{(s-a)^n}$, R = 1.98 cal/moleK

$\mathcal{L}[\int_0^t f(t-\xi)g(\xi)\, d\xi] = \mathcal{L}[f(t)]\, \mathcal{L}[g(t)]$, $\mathcal{L}[\sin(at)e^{-bt}] = \frac{a}{[(s+b)^2+a^2]}$, $\mathcal{L}[\frac{[be^{bt} - ae^{at}]}{(b-a)}] = \frac{s}{(s-a)(s-b)}$ for $a \neq b$, $\ln\frac{\nu_2}{\nu_1} = \frac{U}{R}\{\frac{1}{T_1} - \frac{1}{T_2}\}$.

1 (20 pts) Define the following using one or two sentences, or if appropriate, by an equation
(a) Biot theory
(b) Resonant ultrasound spectroscopy
(c) Durometer
(d) Glassy region
(e) Thermoelastic relaxation

2 (20 pts) Give approximate Young's modulus and tan δ values for each of the following at 20 °C, 10 Hz.
(a) Polymethyl methacrylate (PMMA) (b) Aluminum alloy (c) Wood (d) Quartz (e) Object at desk.

3 Consider indentation of a semi-infinite substrate by a flat ended cylindrical rigid indenter. The elastic solution is given as the following.

$u = F(1-\nu^2)/2RE$, with u as displacement, F as the applied force, R as indenter radius, E as substrate Young's modulus, ν as substrate Poisson's ratio.

(a) (30 pts) Determine the creep compliance if the material is viscoelastic. Assume that the applied force is a step function in time and that the indenter displacement is measured as a function of time. Use the given equation. Assume that the Poisson's ratio is constant in time.

(b) (30 pts) How may this analysis be used to design an experiment? What is the rationale for such an experiment? If the Poisson's ratio actually varies with time, describe how you would modify the analysis. How would the interpretation differ if the substrate were a composite?

(c) (10 pts) Extra credit. Suppose the applied force is a sinusoidal function of time. Determine analytically what viscoelastic properties can be extracted. What might be the advantages or disadvantages of such a procedure?