Given: \( \varepsilon_p = \partial u/\partial r; \) \( \varepsilon_{\phi} = (\partial u/\partial \phi + u_\phi)/r; \) \( \varepsilon_r = \partial u/\partial z; \) \( \varepsilon_{\theta} = \frac{[\partial u/\partial \theta + \partial u/\partial z]/2}{2} \) \( e_m = \frac{[\partial u/\partial r + \partial u/\partial \phi]}{2}; \)

\( e_r = [\partial u/\partial r + (1/r) \partial u/\partial \phi - u_\phi / r]/2, \) \( \sigma_{ij, j} + F_i = \rho a_i; \) \( \varepsilon = E \varepsilon; \) \( \sin(\theta \pm q) = \sin \theta \cos q \pm \cos \theta \sin q. \) \( J^* = 1/E*. \)

\( J^*(\omega) = \frac{2\omega}{\pi} \left[ \begin{array}{c} 1 \int J(\omega) J(\omega) d\omega \end{array} \right] \)

\( J(\omega) - J(\omega) \equiv \frac{2\omega}{\pi} \left[ \begin{array}{c} 1 \int \frac{J(\omega)}{\omega^2 - \omega^2} d\omega, \sigma(\omega) \equiv \int (e(t) - e(t)) \frac{d\tau}{dt} \end{array} \right] \)

\( L[t] = \frac{1}{s^2}, \) \( L[\sigma(t-a)] = e^{-at/s}, \) \( L[\sigma(t-a)] = e^{-at}, \) \( L[\sin(at)e^{-bt}] = \frac{a}{(s+b)^2 + a^2}, \) \( L[\frac{he^{bt} - he^{-at}}{b-a}] = \frac{s}{(s-a)(s-b)} \) for \( a < b. \) \( \ln V_2 = \frac{U}{R} \left( \frac{1}{T_1} - \frac{1}{T_2} \right). \)

Solve four problems only and state which four. Show all logic and state assumptions!!

1 (25 pts) Define the following. One sentence or a diagram or an equation should suffice.
  - (a) attenuation
  - (b) tan \( \delta \)
  - (c) resonance
  - (d) shift factor
  - (e) stretched exponential
  - (f) Boltzmann superposition principle
  - (g) spectrum of relaxation times
  - (h) Debye peak

2 (10 pts) Show that for a linearly viscoelastic material, \( J' = \frac{1}{E' + \tan^2 \delta} \)

(b) (10 pts) What is the physical interpretation of \( J' > E' \) in the context of a stress-strain diagram?

(c) (5 pts) How are \( J' \) and \( E' \) related for an elastic material?

3 (a) (5 pts) Draw the stress strain curve for a linearly viscoelastic solid under sinusoidal strain \( \varepsilon(t) = B \sin \omega t. \)

(b) (5 pts) Suppose the stress is \( \sigma = D \sin(\omega t + \delta). \) What is the meaning of \( \delta? \)

(c) (5 pts) Find the slope of the line from the origin to the point of maximum stress. Hint: write strain in terms of stress and let the stress assume its maximum value; start with \( \sigma(t) = \sigma_{\text{max}} \sin(\omega t). \)

(d) (5 pts) Show \( \sin \delta = A/B. \) \( A \) is the intercept on strain axis. Hint: let \( \omega t = -\delta \) in equation in (b) for the stress.

(e) (5 pts) Draw a stress strain curve for a nonlinearly viscoelastic material under sinusoidal strain.

4 The end deflection \( u \) of an elastic cantilever beam of length \( L, \) Young's modulus \( E \) and cross sectional area moment of inertia \( I \) is given as \( u = 2F/L^3/6EI, \) with \( F \) as the applied force.

Suppose now the beam is linearly viscoelastic and that any needed viscoelastic properties are known. If the force has a time dependence \( F(t), \) determine the time dependence \( u(t) \) of the deflection.

5 Consider \( E(t) = A + Be^{-bt} \) with \( A, B, b \) as constants.

(a) (10 pts) If a relaxation experiment were conducted on your earplug material, do you expect it would follow the above equation? Explain why or why not.

(b) (15 pts) Suppose a material with the given \( E(t) \) is subject to step strain \( \varepsilon(t) = [\varepsilon_0 + \varepsilon_1 e^{-at}]H(t), \) with \( a \) as a constant and \( \varepsilon_0, \varepsilon_1 \) as constants. Hint: assume strain starts just after zero so no surface terms.

Determine the stress \( \Omega(t). \)