Brodt, M. and Lakes, R. S., "Composite materials which exhibit high stiffness and high viscoelastic damping", adapted from *J. Composite Materials*, 29, 1823-1833, (1995).

ABSTRACT

Composite micro-structures are studied, which give rise to high stiffness combined with high viscoelastic loss. We demonstrate that such properties are most easily achieved if the stiff phase is as stiff as possible. Incorporation of a small amount of damping in the stiff phase has little effect on the composite damping. Experimental results are presented for laminates consisting of cadmium and tungsten and of InSn alloy and tungsten. The combination of stiffness and loss (the product E tan δ) exceeds that of well-known materials.

1. INTRODUCTION

Viscoelastic materials are of use in the damping of vibration and in the absorption of sound waves. In some applications it is desirable to use materials which are stiff enough to carry out a structural role, and which also exhibit significant mechanical damping. However most stiff materials are low in damping and most high-damping materials are compliant. Properties of some representative materials are shown in Fig. 1. In some applications a layer of polymer is cemented to a metal to increase the damping. The attainable damping in many configurations is limited by the fact that polymers, though they may exhibit high damping, are not very stiff. The loss tangent, tan δ, is used throughout as the measure of viscoelastic damping, or loss.

Chen and Lakes [1] have shown that composite materials can, in principle, exhibit high stiffness and high loss. An essential element in achieving a stiff and lossy composite is *non-affine deformation*, in which the strain field is highly inhomogeneous on the scale of micro-structural elements. One way to achieve this is via re-entrant structure which gives rise to a negative Poisson's ratio [2,3]. Among composites with simple structure, a Reuss-type composite is most favorable for this purpose, since the deformation is highly inhomogeneous. The Reuss geometry is considered unfavorable in terms of stiffness for a given volume fraction of the stiff phase, however a stiff Reuss composite can be made if the volume fraction of the compliant phase is made very small.

We consider as a goal the development of materials which exhibit a Young's modulus of 70 GPa, corresponding to aluminum, and a loss tangent of 0.06, representative of a glassy or of a crystalline polymer (which would have a Young's modulus of 3 GPa or less).

2. ANALYSIS: STIFFNESS-LOSS MAPS

The consequences of several composite variables are explored numerically with the aid of stiffness-loss maps. The rationale is to develop a strategy for producing composite materials with high stiffness and high loss. The analysis in ref. [1] showed that a Reuss or Reuss-like structure is favorable in that regard; it remains to be determined what are the optimal characteristics of the phases. The maps are constructed from the equations for the stiffness of an elastic composite, with the aid of the dynamic correspondence principle of linear viscoelasticity [4,5]. Each elastic modulus is replaced with a complex dynamic modulus in the analysis. In particular, for elastic phases, the Voigt (or uniform strain) relation is

$$E_c = E_1V_1 + E_2V_2, \quad (1)$$

in which E_c, E_1 and E_2 refer to the Young's modulus of the composite, phase 1 and phase 2, and V1 and V2 refer to the volume fraction of phase 1 and phase 2 with $V_1 + V_2 = 1$. We apply the correspondence principle to convert the elastic relations to a viscoelastic relation (for loading which is sinusoidal in time) by replacing the Young's moduli E by $E^*(i\omega)$ or E^*, in which ω is the angular frequency of the harmonic loading. This procedure gives

$$E_c^* = E_1^* V_1 + E_2^* V_2, \tag{2}$$

with $E^* = E' + i\,E''$ and the loss tangent $\tan\delta = E''/E'$.

In the Reuss model each phase experiences the same stress. For elastic materials,

$$\frac{1}{E_c} = \frac{V_1}{E_1} + \frac{V_2}{E_2} \tag{3}$$

Again using the correspondence principle, the viscoelastic relation becomes

$$\frac{1}{E_c^*} = \frac{V_1}{E_1^*} + \frac{V_2}{E_2^*}. \tag{4}$$

Separating the real and imaginary parts of E_c^*, the loss tangent of the composite $\tan\delta_c$ is obtained numerically.

The curves for the Voigt and Reuss composites enclose a region in the map, but these curves are not bounds. Bounds for the complex bulk modulus have been presented [6]. We remark that the Voigt and Reuss curves appear almost identical to the Hashin-Shtrickman stiffness bounds on a stiffness-loss map [1] even though they can be very different when plotted as functions of volume fraction. For the purposes of this study, the Voigt and Reuss composites are considered since they are both simple and are realizable physically. We are concerned with the low frequency range, well below any resonances in the laminae, therefore more complex modeling is unnecessary. Moreover, these composites exhibit viscoelastic behavior which, in the map, is close to that of the bulk modulus bounds for many constituent values. Fig. 2 shows the effect of increasing the stiffness of the stiff phase, from 200 GPa, corresponding to steel, to 400 GPa, corresponding to tungsten. For both, $\tan\delta$ is assumed to be 0.001, a value representative of results reported in the literature. The more compliant phase is assumed to have $E = 31$ GPa, and $\tan\delta = 0.12$. Observe that use of a stiffer stiff phase facilitates achievement of a composite which is both stiff and lossy. Fig. 3 shows the effect of an increase of the damping ($\tan\delta$) of the stiff phase, from 0.001 to 0.01. The lower value is representative of steel, and the higher value is representative of gray cast iron. Observe that there is a minimal effect on the stiffness-loss map in the upper right hand region which would correspond to stiff and lossy materials. Consequently there is not much benefit in using a stiff phase with higher damping, such as gray cast iron. Fig. 4 shows the effect of changing the properties of the lossy phase. As would be expected, a lossy phase which is as stiff as possible gives rise to a composite with both high stiffness and high loss. Moreover, both the Reuss and Voigt curves are convex to the right when the lossy phase is stiff. Therefore composite materials with several kinds of structures can give rise to favorable combinations of stiffness and loss. Care is required in the choice of the phase geometry. It may be tempting to incorporate soft, compliant inclusions in a stiff, strong matrix, since such a configuration is simple. However, the *shape* of the curve in the stiffness-loss map is very unfavorable for such a geometry; it is convex to the lower left [1]; see also the lower left curve in Fig. 4.

3. MATERIALS AND METHODS

Materials were chosen in view of the above analysis. Tungsten was chosen as the stiff phase since it has a relatively high Young's modulus (400 GPa), is machinable and it is readily available in a variety of forms. We remark that ceramics such as alumina (Al_2O_3) also are quite stiff. The high damping phase was chosen as cadmium for one laminate, and indium-tin eutectic alloy for the other. The cadmium and InSn alloy are described elsewhere [7,8]. Considerable effort was required to identify appropriate high damping phases. For example, lead, which is commonly thought of as offering high damping, actually exhibits a $\tan\delta$ of 0.01 or less, combined with low stiffness. The metals were cut with a low-speed diamond saw, and the surfaces were finished by lapping with fine abrasives. Laminates were prepared by adhesive bonding using a cyanoacrylate adhesive. A single volume fraction was chosen with the aim of achieving a composite stiffness exceeding that of aluminum, combined with a loss tangent exceeding 0.06; it is not the purpose of the study to verify the volume fraction dependence of the Reuss model. The laminates were prepared in rod form 3.1 mm in diameter to facilitate experimental study. The W-Cd laminate had a volume fraction of 50%

tungsten, and had three laminae of each constituent. The first W-InSn laminate had three laminae of tungsten, 8 mm thick, and two of indium-tin, with a volume fraction of 95% tungsten. In the second W-InSn laminate, the laminae were about three times thicker.

Viscoelasticity studies were performed in bending using a modified version [9] of the apparatus of Chen and Lakes [10], at ambient temperature (23°C). The instrument was developed with the capacity to do either torsion or bending studies, and allows a wide frequency range in torsion from 10^{-6} Hz to about 10 kHz. The capability for high frequency experiments comes in part from the fact that minimal inertia is attached to the specimen. The present version of the instrument incorporates improved phase resolution, sufficient for metallic specimens of moderately high loss. One end of the specimen was cemented (with a cyanoacrylate adhesive) to the rigid framework and a magnetic disk and mirror were glued to the other end. This specimen configuration is essentially fixed-free since the support rod for the specimen is 12.5 mm in diameter (compared with 3.1 mm for the specimen) and is made of tungsten ($E \approx 400$ GPa); the apparatus is orders of magnitude stiffer than the specimens studied. A sinusoidal voltage from a digital function generator was applied to the Helmholtz coil which in turn caused either an axial torque or a bending moment, depending upon the orientation of the coil with respect to the magnet, on the magnetic disk, and thus the specimen. The experiments reported here were done in bending at frequencies well below the first resonance. The bending moment vector was parallel to the laminae so that the stress was orthogonal to the lamina planes. Light from the laser was reflected from the specimen's mirror to a split-diode light detector connected to a differential amplifier. The detector provides a linear voltage response to angular displacement. The phase angle between torque and angular displacement was measured via a lock in amplifier or by determining the width of a digitized Lissajous figure, as described earlier [9]. The loss tangent was inferred from the measured phase angle. The viscoelastic loss tangent ($\tan \delta$) is the tangent of the phase angle between stress and strain for sinusoidal loading in time. Phase resolution was about 0.001 radian in the sub-resonant regime; resolution is not an issue in the present study in view of the large phases involved. Parasitic losses such as air damping, can be important in studies of low-loss materials, particularly in bending. Study of a specimen of low loss aluminum alloy indicated parasitic losses to be negligible (< 0.001) at the low frequencies involved in this study.

4. RESULTS AND DISCUSSION

Results are shown in Fig. 6 and 7. The laminates exhibit a very unusual combination of high stiffness and high loss. Even so, observed stiffness is less than the value predicted theoretically with a two-phase Reuss model. The difference is attributed to the compliance of the glue joint, which could not be made infinitesimally thin. Specifically, in W-InSn, for a volume fraction of tungsten of 0.95, two-phase Reuss theory, based on experimental values at 1 Hz, $E = 400$ GPa and $\tan \delta = 0.007$ for tungsten and $E = 20.9$ GPa and $\tan \delta = 0.09$ for InSn, predicts for the laminate, $E = 210$ GPa and $\tan \delta = 0.048$. Experimentally, $E = 86$ GPa and $\tan \delta = 0.05$. The experimental laminate loss tangent is in agreement with the theoretical value, but the experimental stiffness is lower by a factor of 2.4. Now at 0.01 Hz, $E = 163$ GPa and $\tan \delta = 0.20$ are predicted and $E = 68$ GPa and $\tan \delta = 0.24$ are observed. Again the loss tangents are in reasonable agreement but the observed stiffness is lower by a factor of 2.4. A three-phase Reuss analysis was then used. The mechanical properties of cured cyanoacrylate cement are not available, but representative polymer values of $E = 1$ GPa and $\tan \delta = 0.05$ were chosen for illustrative purposes. Under these assumptions, if the six glue joints were 32μm thick, the experimental reduction in stiffness would be accounted for. We remark that the experimental results presented here are at low frequency, well below the resonant frequency of the specimen or of its constituents. At such low frequencies, the simple Reuss model suffices; however at higher frequency, resonances in the laminae would occur and a more complex theoretical approach would be needed.

A second laminate was prepared, with thicker laminae. $E = 161$ GPa with $\tan \delta = 0.096$ was achieved at 0.1 Hz, and $E = 129$ GPa with $\tan \delta = 0.2$ at 0.01 Hz. These values are closer to the ideal response of a two-phase laminate, as a result of the thinner layers of cement in comparison with the laminae.

To fully attain the limiting stiffness and loss for a true two-phase system, compliance due to adhesive joints must be eliminated or reduced. Diffusion bonding is a possibility, but would be difficult in view of the disparity in melting points of the constituents. Another possibility would be to cast particles of the stiff phase in a high loss matrix.

The Reuss geometry used in tungsten indium-tin laminates gives a favorable combination of stiffness and viscoelastic loss. However such a geometry does not confer high strength, except in compression. Higher strength may be achieved by stacking laminae, of different orientation [11], each of which contains two phases, as is commonly done for other purposes in fibrous composites.

5. CONCLUSION

A combination of high stiffness and high mechanical damping is attainable in laminates of a stiff phase such as tungsten and a phase with high loss and moderate stiffness such as cadmium or indium-tin alloy.

ACKNOWLEDGMENT

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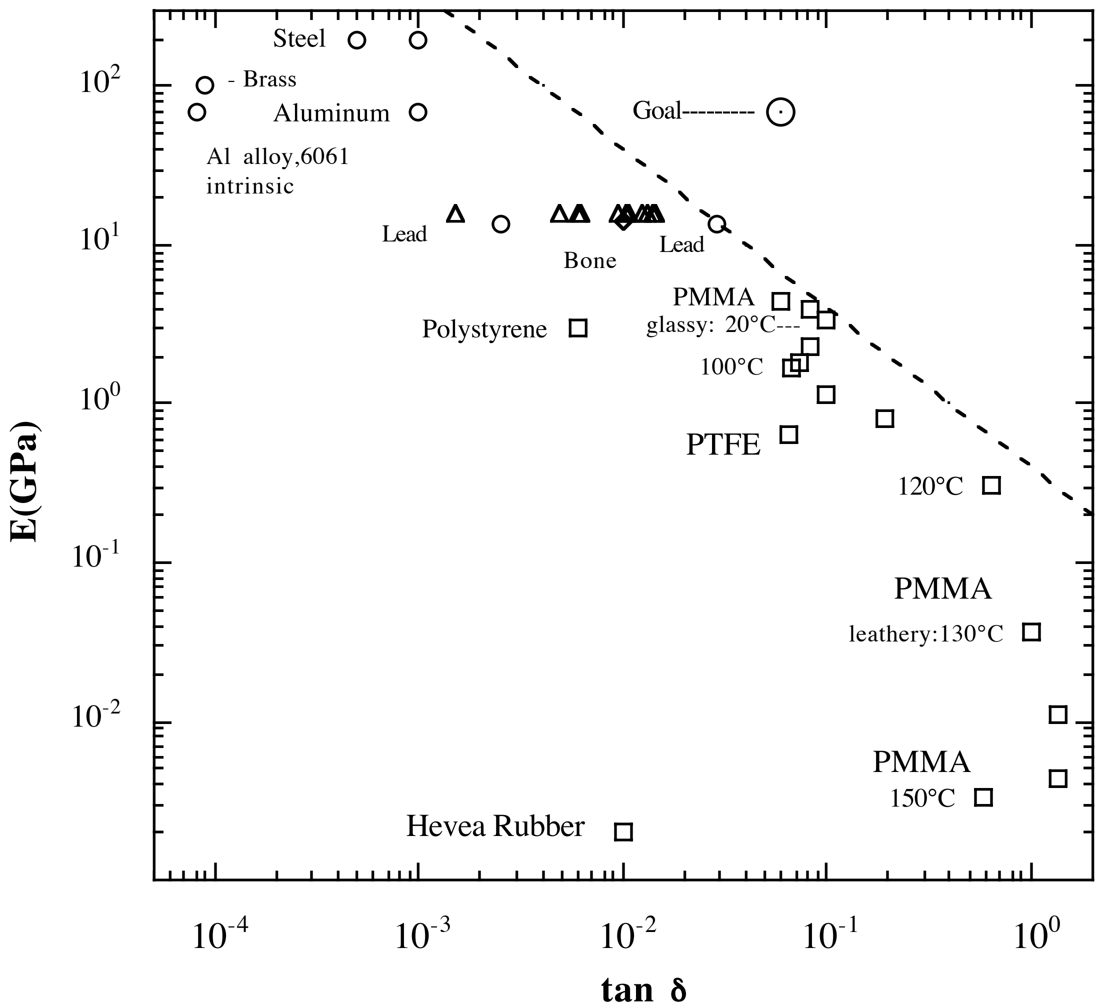
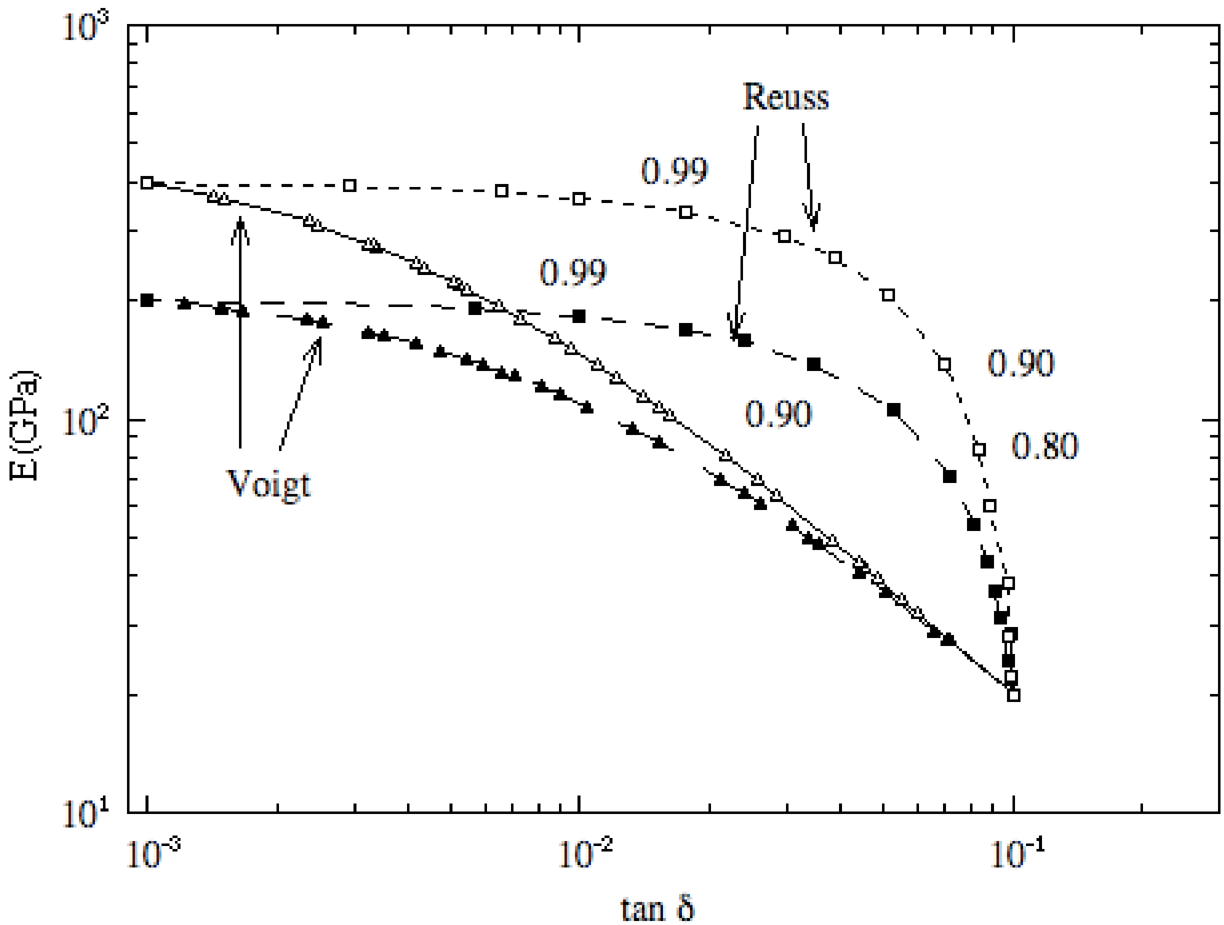
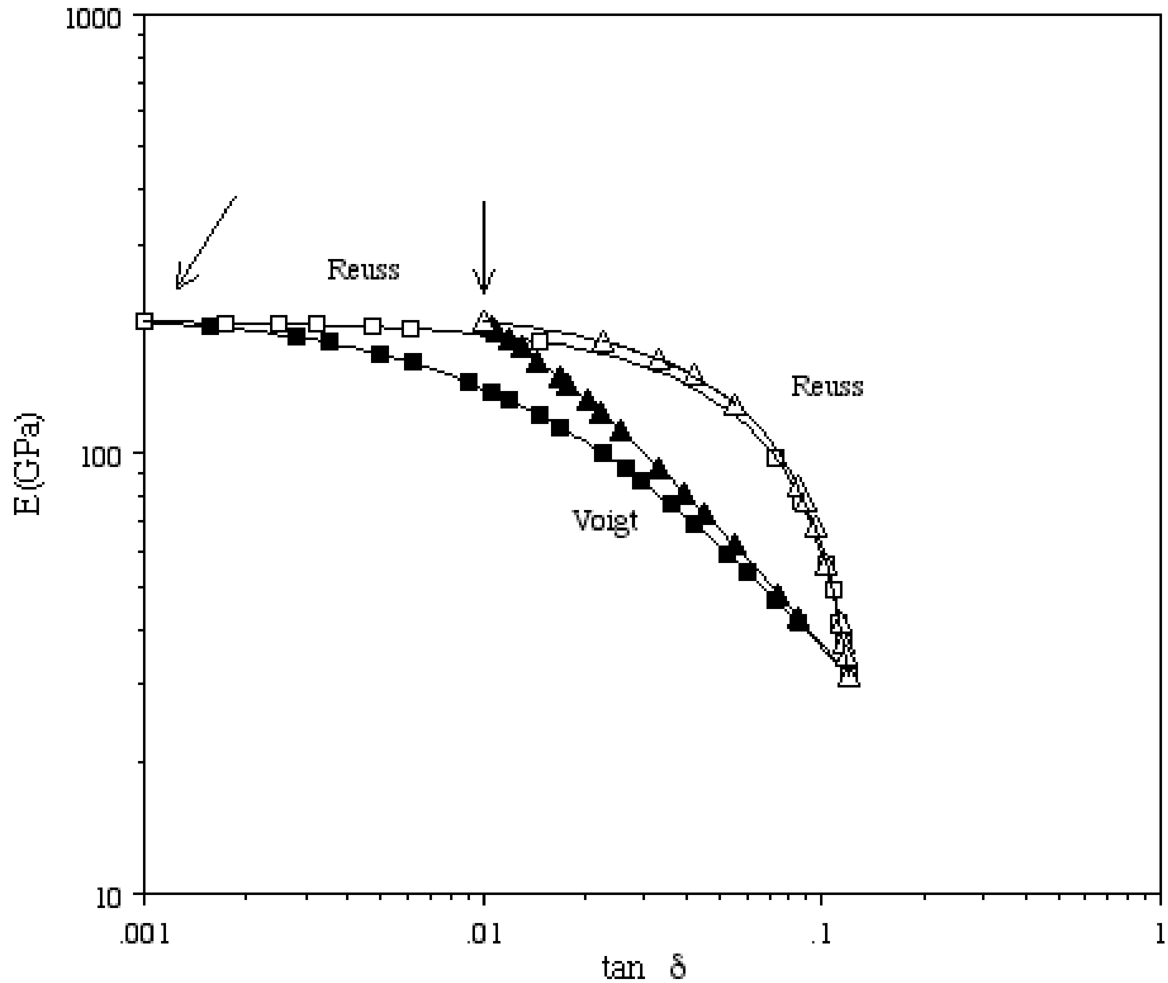
Figures



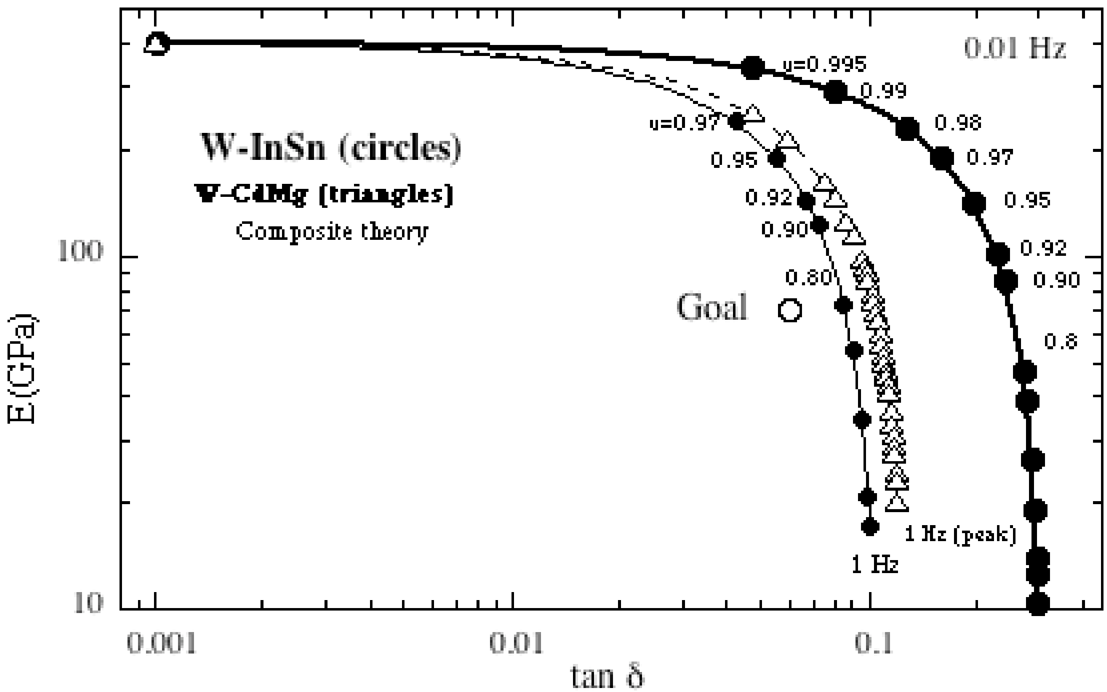
1. Stiffness-loss map for some common materials. Values for PMMA converted from torsion data of Iwayanagi, S. and Hideshima, T., "Low frequency coupled oscillator and its application to high polymer study" J. Phys. Soc. Jpn, 8, 365-358, 1953, at various temperatures and less than 1 Hz. Other values are for room temperature, at various frequencies.



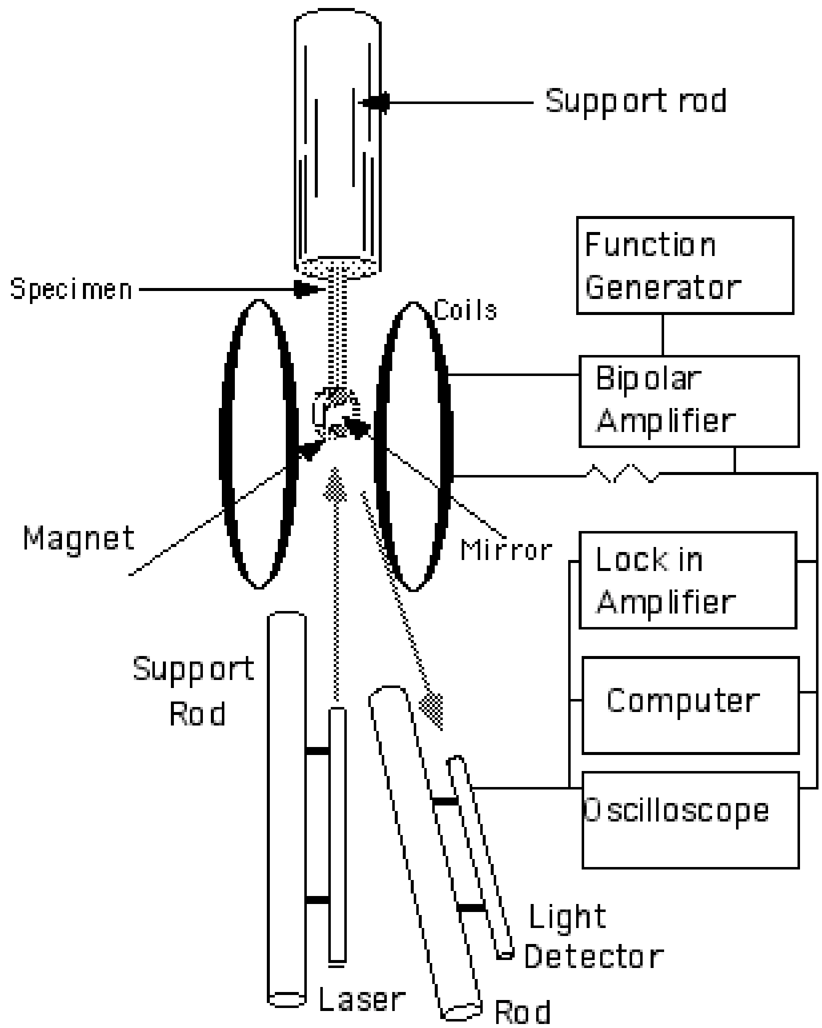
2. Use of a stiffer stiff phase (400 GPa vs 200 GPa) facilitates achievement of a stiff and lossy composite. Theoretical stiffness-loss maps for Voigt and Reuss composite structures. Stiff phase is assumed to have a loss tangent of 0.001.



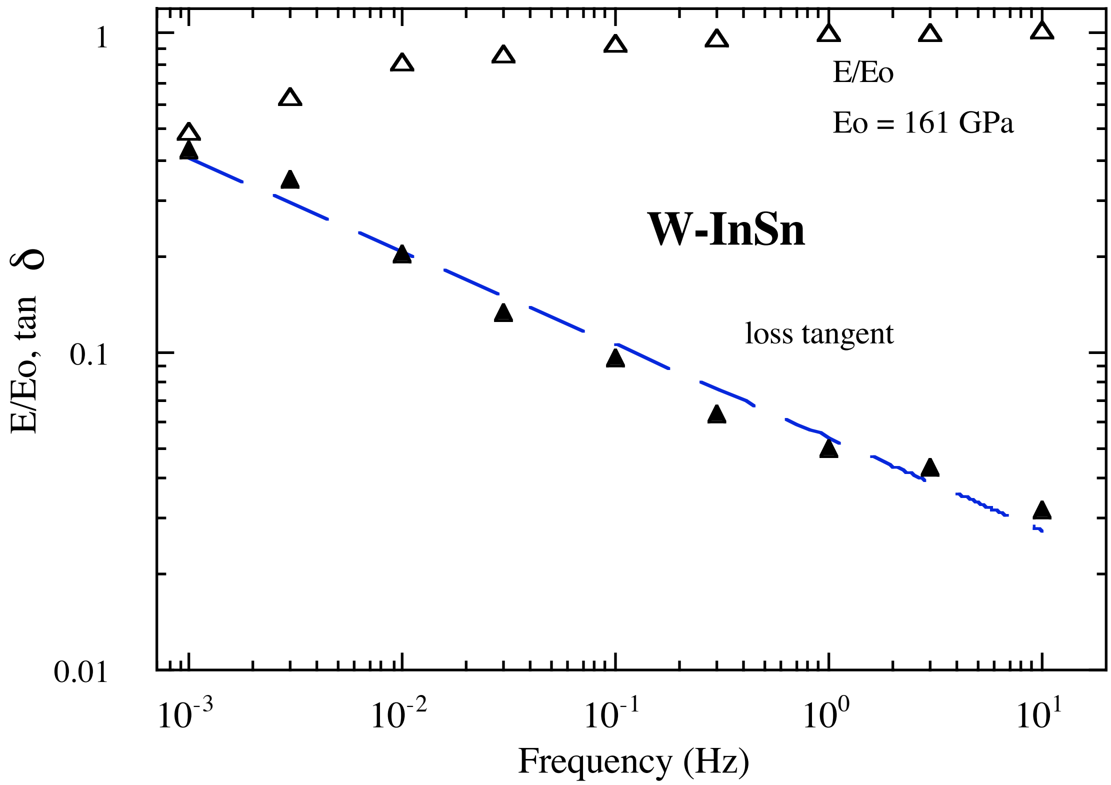
3. Use of a stiff phase with a moderately high loss tangent (0.01 vs 0.001) has a minimal effect on the stiffness-loss map in the upper right hand region. Theoretical stiffness-loss maps for Voigt and Reuss composite structures. Stiff phase is assumed to have a Young's modulus of 200 GPa. The lossy phase is assumed to have E = 31 GPa and tan δ = 0.12. Points for 100% stiff phase are indicated by arrows.



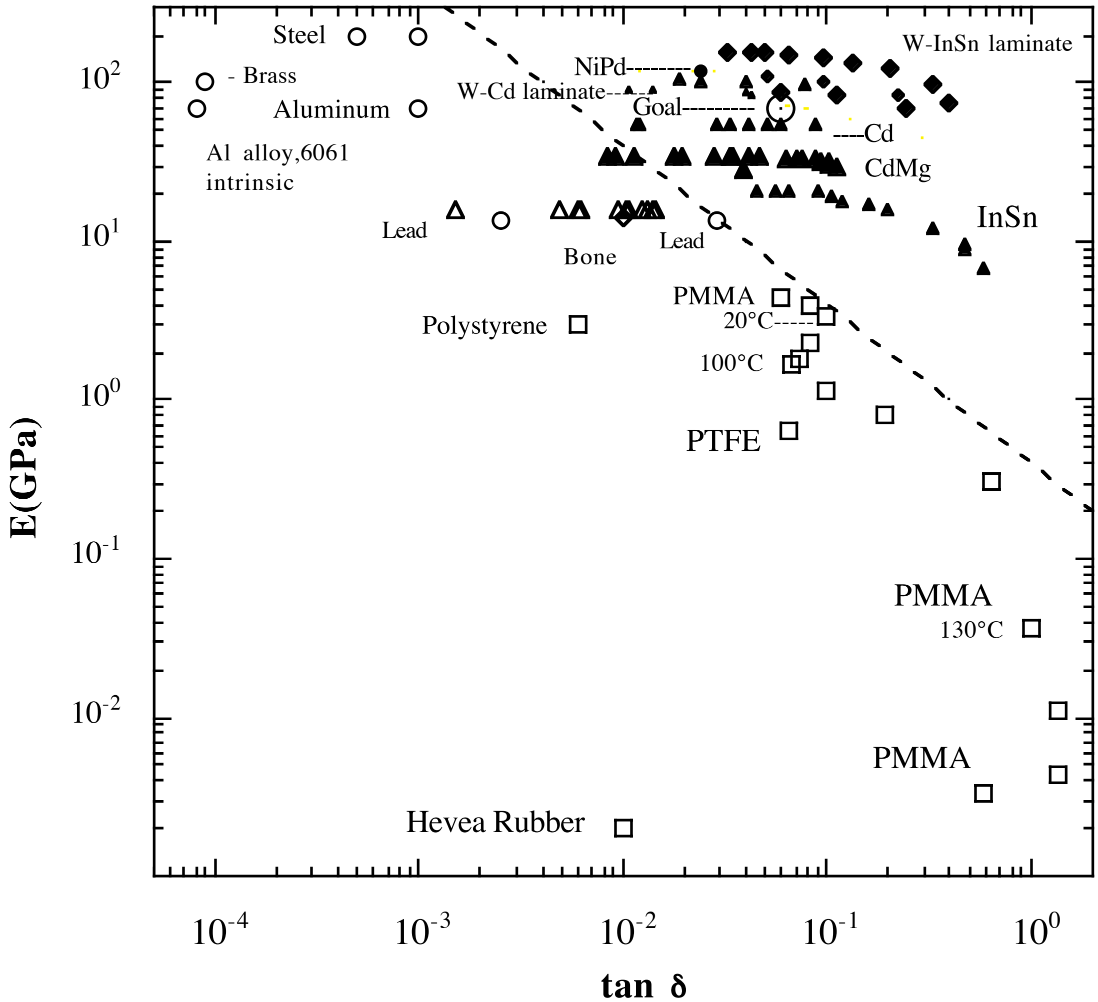
4. Use of a lossy phase which is relatively stiff causes both the Voigt and Reuss curves to be convex to the right. Theoretical stiffness-loss maps for Voigt and Reuss composite structures. Stiff phase is assumed to have a loss tangent of 0.001.



5. Schematic diagram of the apparatus.



6. Experimental results: bending stiffness and damping vs frequency for W-InSn laminate.



7. Experimental results: stiffness-loss map showing the behavior of the present laminates in relation to other materials.