MSE 541 *Final Quiz*

Given: $\frac{E}{E_s} = C_1[\frac{\rho}{\rho_s}]$, $\frac{E}{E_s} = C_2[\frac{\rho}{\rho_s}]^2$, $\frac{E}{E_s} = C_3[\frac{\rho}{\rho_s}]^3$, $\nu_{31} = \nu_s$, $\nu_{21}^{regular} = 1$, $\nu_{iso} = \frac{1}{3}$, $\frac{\sigma_e^{regular}}{E_s} = 5.2[\frac{t}{L}]^3$, $\frac{\sigma_e^f}{E_s} = 0.05[\frac{\rho}{\rho_s}]^2$, $\sigma_{ij} = C_{ijkl}\epsilon_{kl}$, $\epsilon_{ij} = J_{ijkl}\sigma_{kl}$, $G_c = G_1V_1 + G_2V_2$, $J_c = J_1V_1 + J_2V_2$, $G_L = G_2 + \frac{V_1}{\frac{1}{G_1 - G_2} + \frac{6(K_2+2G_2)V_2}{5(3K_2+4G_2)G_2}}$, $K_L = K_2 + \frac{V_1(K_1-K_2)(3K_2+4G_2)}{(3K_2+4G_2)+3V_2(K_1-K_2)}$, $C_{1111} = E\frac{1-\nu}{(1+\nu)(1-2\nu)}$, $E = 2G(1+\nu)$, $\nu = \frac{3B-2G}{6B+2G}$, $A_c = \pi r^2$, $v_s = \frac{4}{3}\pi r^3$, $F_{crit} = \frac{n^2\pi^2 E_s I}{L^2}$, Epoxy resin, $E = 3.6$ GPa, $\sigma_{ult} = 90$ MPa, $\rho = 1.25$ g/cm^3; Boron fiber, $E = 400$ GPa, $\sigma_{ult} = 3.5$ GPa, $\rho = 2.3$ g/cm^3. Boron-epoxy unidirectional composite, $E_L = 210$ GPa, $E_T = 21$ GPa, $G_{LT} = 7$ GPa, $\sigma_{ult}^L = 2.6$ GPa, $\rho = 2.0$ g/cm^3. Graphite-epoxy unidirectional composite, $E_L = 160$ GPa, $E_T = 11$ GPa, $G_{LT} = 6.4$ GPa, $\sigma_{ult}^L = 1.72$ GPa, $\sigma_{ult}^T = 42$ MPa, $\tau_{ult} = 95$ MPa, $\rho = 1.6$ g/cm^3, $u = PL^3/3EI$, $I = t^4/12$

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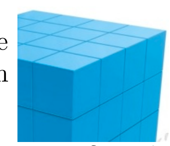
G

Solve ***five*** problems only and state which five. **Show all logic**. Enjoy!

1. (20 points, 2 each) Define *ten* of the following via a sentence, equation or diagram.
(a) Voigt (b) open cell foam (c) hierarchical (d) osteon (e) orthotropic (f) isotropic (g) quasi-isotropic laminate (h) piezoelectric (i) chiral (j) creep (k) relaxation modulus (l) Hashin-Shtrikman formula (m) binder (n) collagen

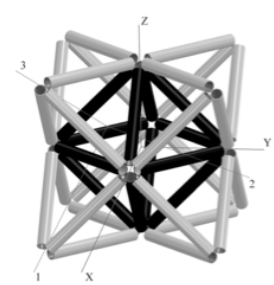
2. (20 points, 10 each) Consider stiffness of cellular solids. The following may be regarded as known. For plate elements of width and length L and thickness t, $[\frac{\rho}{\rho_s}] = C_p\frac{t}{L}$. For rib elements of length L and thickness t, $[\frac{\rho}{\rho_s}] = C_r[\frac{t}{L}]^2$.
(a) Show that for honeycomb compressed out of plane, the Young's modulus is given by $\frac{E}{E_s} = [\frac{\rho}{\rho_s}]$.
(b) Show that for open cell foam, $\frac{E}{E_s} = C_2[\frac{\rho}{\rho_s}]^2$. Do not determine the proportionality constant.

3. (20 points) Consider shaped particulate inclusions. Constituent moduli are assumed known.



(a) (7 pt) Consider a particulate composite modeled as a packing of cubical inclusions of side a separated by a thin layer of matrix of thickness t. Obtain an approximate analytical expression for the on-axis compressive modulus. Neglect Poisson effects. Consider Reuss, Voigt.
(b) (7 pt) How can high inclusion concentration be achieved if the inclusions are not cubes or prisms? Make a sketch.
(c) (6 pt) Give a technological example of a composite with high inclusion concentration. Explain how the high concentration is achieved and how it benefits performance. Similarly give a biological example.

4. (20 points) Consider an octet truss lattice of slender straight ribs of length L. Density is low enough that stress around joints and the mass of joints can be neglected, and that elastic buckling is the failure mechanism in compression.



(a) (7 pt) Show that the compressive strength is $\frac{\sigma_e^f}{E_s} \propto [\frac{\rho}{\rho_s}]^2$. Do not determine the proportionality constant. The ribs are solid rods of radius r.
(b) (7pt) Suppose the ribs are made a factor of two thicker and are made tubular (hollow) to maintain the same weight. The length is unchanged. By what factor does the lattice Young's modulus change? Why?
(c) (6 pt) Suppose the ribs are made a factor of two thicker and are made tubular (hollow) to maintain the same weight. The length is unchanged. By what factor does the lattice compressive strength change? Why?

5. (20 points) Referring to your lab reports (either one) answer *one* of the following.
(a) Why is the initial portion of the compressive stress-strain plot curved even if the material is behaving linearly at small strain? How does this effect differ if the material is a foam or a composite? How does one obtain correct interpretation of modulus from slope in your present experiment? How could one design an experiment to eliminate the initial curve?
(b) How does delamination manifest itself in a stress-strain curve? How does failure of the bond between phases manifest itself? How is your delamination result pertinent to applications?

6. (20 points) Consider a unidirectional boron-epoxy composite. The volume fraction of fibers is 0.5.
(a) (7 pt) Determine the Young's modulus in the longitudinal direction.
(b) (7 pt) Determine a lower bound on the Young's modulus in the transverse direction.
(c) (6 pt) What is the density of the composite?