PHILOSOPHICAL MAGAZINE LETTERS, 2001, VOL. 81, NO. 2, 95–100

Extreme damping in compliant composites with a negative-stiffness phase

R. S. LAKES†

Department of Engineering Physics, Materials Science Program and Rheology Research Center, University of Wisconsin-Madison, 147 Engineering Research Building, 1500 Engineering Drive, Madison, Wisconsin 53706-1687, USA

[Received in final form 5 October 2000 and accepted 6 October 2000]

ABSTRACT

Compliant composite unit cells were made with negative stiffness constituents. Flexible silicone rubber tubes were incorporated in a post-buckled condition to achieve negative stiffness. Large peaks in the mechanical damping $\tan\delta$ were observed in these systems. Maximum damping was orders of magnitude in excess of the material damping of the silicone rubber.

§ 1. INTRODUCTION

Negative stiffness entails a reversal of the usual directional relationship between force and displacement in deformed objects. It is not illegal (does not violate any physical law), but an isolated object with negative stiffness is unstable. Negative stiffness can be achieved as follows. A column constrained in a buckled 'S'-shaped configuration is in unstable equilibrium (Bazant and Cedolin, 1991). By pressing laterally on the column one can cause it to snap through. The column can be stabilized by a lateral constraint. One can easily verify the properties of the buckled column with the aid of a flexible plastic ruler. Tubes in the post-buckling regime exert decreasing force with an increase in deformation, and hence negative incremental stiffness. Flexible tetrakaidecahedra (Rosakis et al. 1993) were considered as models of single cells in foams and deformed in compression under displacement control. They exhibit a non-monotonic force–deformation relation, hence they exhibit negative stiffness over a range of strains.

Negative stiffness differs from negative Poisson's ratio in that negative Poisson's ratio is possible with positive, but unusual, combinations of stiffness. Poisson's ratio ν is defined as the negative transverse strain of a stretched or compressed object divided by its longitudinal strain. For most solids, ν is between 0.25 and 0.33. Foams (Lakes 1987, 1993) with Poisson's ratio as small as -0.7 have been made. Young's modulus E and the shear modulus G are related to Poisson's ratio in isotropic materials by relations such as $E = 2G(1+\nu)$. The 'allowable' range of Poisson's ratios, $-1 < \nu < 0.5$, corresponds to the requirement that the moduli be positive for stability of an unconstrained (surface traction boundary condition in the language of elasticity) block of material.

The current investigation experimentally explores viscoelasticity in compliant composite systems with negative-stiffness constituents.

† Email: lakes@engr.wisc.edu.

Philosophical Magazine Letters ISSN 0950–0839 print/ISSN 1362–3036 online © 2001 Taylor & Francis Ltd
http://www.tandf.co.uk/journals
DOI: 10.1080/09500830010015332

§ 2. Analysis

The stiffness of a linearly elastic two-phase composite for a given volume fraction of one phase is bounded by the Voigt and Reuss composites (Paul, 1960). The Voigt upper bound formula, also called the rule of mixtures, is

$$E_c = E_1 V_1 + E_2 V_2, \tag{1}$$

in which E_c, E_1 and E_2 refer to Young's moduli (stiffness) for the composite, phase 1 and phase 2 respectively, and V_1 and V_2 refer to the volume fraction of phase 1 and phase 2 respectively with $V_1 + V_2 = 1$. The Voigt formula corresponds to a laminate with the laminae aligned with the compressive load. The Reuss structure is aligned perpendicular to the direction of the load so that each phase experiences the same stress. The Reuss lower bound formula is

$$\frac{1}{E_c} = \frac{V_1}{E_1} + \frac{V_2}{E_2}. \tag{2}$$

More restrictive bounds were presented by Hashin and Shtrikman (1963) for isotropic linearly elastic composites. Bounds were also formulated by Gibiansky and Milton (1993) and Gibiansky and Lakes (1993) for viscoelastic composites in a stiffness–loss map (a plot of $|E^*|$ versus $\tan\delta$). For a viscoelastic material, the moduli and compliances are complex: $E^* = E' + iE'' = E'(1 + i\tan\delta)$, with $E' = \mathrm{Re}(E^*)$ and $\tan\delta \equiv [\mathrm{Im}(E^*)]/[\mathrm{Re}(E^*)]$; δ is the phase angle between the stress and strain sinusoids. It is tacitly assumed in the Hashin–Shtrikman bounding analyses that the strain energy density of all phases is positive, corresponding to positive stiffness.

In this work we relax that assumption. The Voigt, Reuss and Hashin–Shtrikman formulae cease to be bounds if stiffness is negative, but they still correspond to realizable composites. To examine the effect of negative stiffness constituents, write the Reuss equation (2) in terms of the compliance $J = 1/E$ rather than the stiffness E.

$$E_c = \frac{1}{J_c} = \frac{1}{J_1 V_1 + J_2 V_2}. \tag{3}$$

One phase is assumed to have a negative stiffness and hence a negative compliance. This compliance may be added to a positive compliance of similar magnitude to obtain a compliance which is very small, tending to zero. The corresponding elastic composite stiffness exhibits a singularity. The form of equation (3) resembles that of a resonance. It is not resonant since the opposing terms in the denominator arise without inertia. Elastic composites with particulate and coated sphere structure give rise to similar singularities and they can be stable (Lakes and Drugan 2000). For a viscoelastic composite, the moduli are complex; figure 1 shows that the composite mechanical damping $\tan\delta$ is predicted to become singular when the positive and negative stiffness values are balanced. The experiments presented here involve flexible tubes in the post-buckling regime of behaviour, shown schematically as insets in figures 2 and 3.

§ 3. Methods

Composite unit cells were fabricated by adhesively bonding silicone rubber tubes with silicone cement, in a Reuss configuration. The configuration, with one tube and two end pieces 10 mm long, is shown in the insets in figure 3; the long tube segment was 40 mm long. Each tube was 10 mm in outside diameter and 6 mm in inner

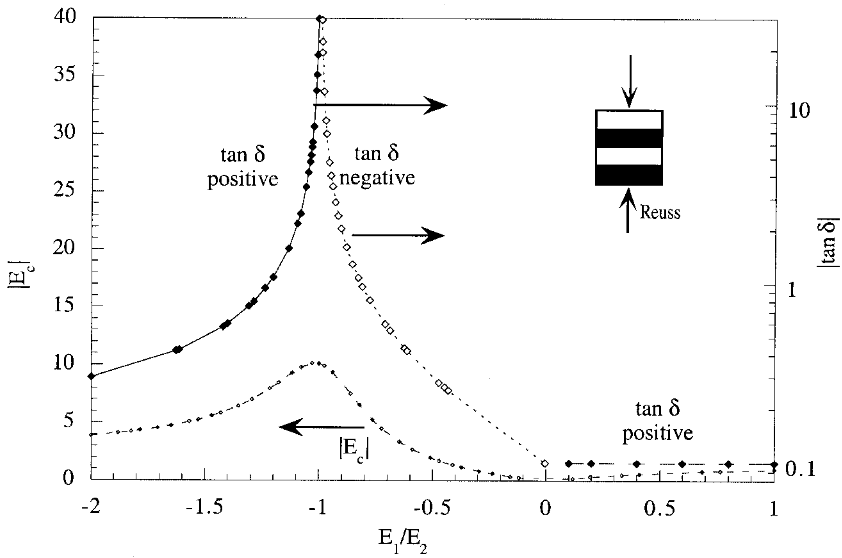

Figure 1. Theoretical modulus $|E_c|$ and damping of a Reuss composite. $\tan\delta = 0.1$ is assumed for the material.

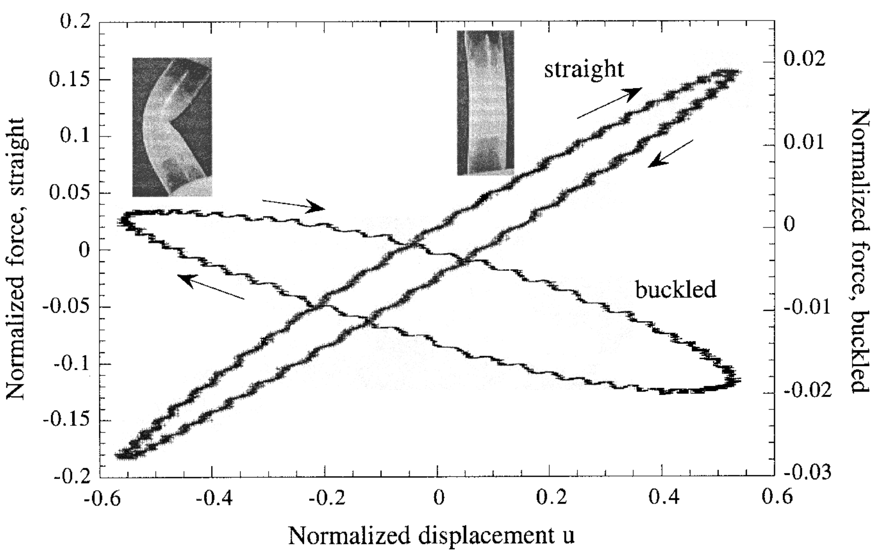

Figure 2. Load–deformation plots for straight and post-buckled tubes under displacement history which is sinusoidal in time at 1 Hz.

diameter. The stability of composites containing many such cells is discussed in §4. The nonlinear post-buckling characteristics of the silicone tubes were examined using a configuration intended to isolate the effect of the kink formed during buckling. Tubes 40 mm long were provided with hard rubber plugs 10 mm long, press-fitted into the lumen and cemented with silicone (insets in figure 2). These were used to examine the post-buckled condition in as pure a form as possible.

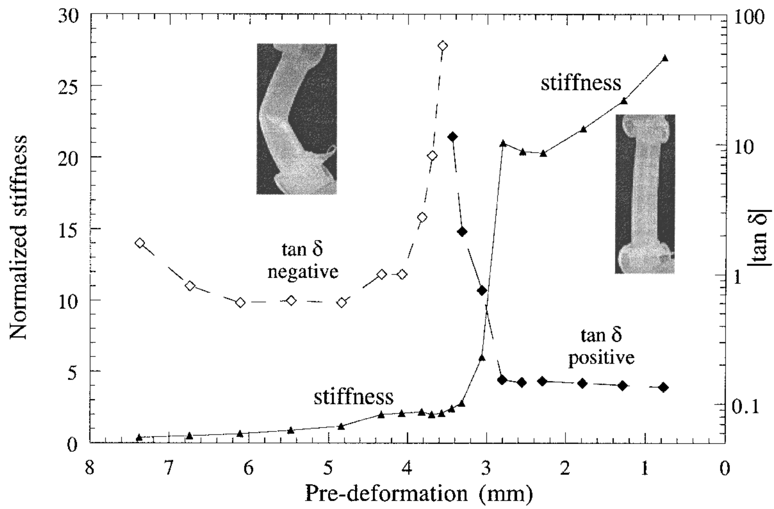

Figure 3. Experimental viscoelastic behaviour of a compliant silicone tube composite unit cell at room temperature.

Material property testing was done with a servohydraulic (MTS type 976) testing system under displacement control. A load cell (Sensotec 41-571) of 100 lb (445 N) capacity was used. Sinusoidal waveforms at 1 Hz were input in displacement control. The amplitude was 0.127 mm. Both the magnitude and the phase of the force response were measured using a lock-in amplifier (Ithaco, type 3961B). To control or tune the effective stiffness of the tube elements, their nonlinear post-buckling characteristic was utilized. Tuning of the effective stiffness was achieved by varying the dc component of displacement, and hence the pre-strain.

§ 4. Results and discussion

The viscoelastic properties of a tube subjected to sinusoidal load while straight and in the post-buckled condition are plotted in the Lissajous figures in figure 2. The straight tube exhibits linearly viscoelastic behaviour with a positive stiffness, as indicated by the positive slope in the load-deformation diagram, and a damping $\tan\delta = 0.12$. Although the response is nonlinear over large deformations, small sinusoidal oscillations about a selected centre point give rise to a linearly viscoelastic load deformation characteristic. The same tube in the post-buckling regime exhibits a negative stiffness as indicated by the negative slope of the corresponding Lissajous figure. Ripples in the curves are due to feed-through noise in the testing machine. In both cases the load–deformation curve is traversed in a clockwise direction as shown by the arrows, as is expected in a passive system with no internal source of power. For the post-buckled tube, the phase angle between load and deformation exceeds 90°, so $\tan\delta$ is negative. According to bounding theorems on restrictions upon viscoelastic behaviour (Christensen 1972) the assumption of a non-negative rate of dissipation of energy gives a non-negative imaginary part to the stiffness, $E'' \geqslant 0$. In the present results, $E' < 0$ and $\tan\delta < 0$; so $E'' > 0$. If one assumes both a non-negative stored energy and a non-negative rate of dissipation of energy, then $E' \geqslant 0$. In the experiments, the 'initial' state has some stored energy. Therefore a negative stiffness E' is not inconsistent with these theorems.

The viscoelastic properties of a Reuss (series) tube composite unit cell are plotted in figure 3. The damping $\tan\delta$ attains a large peak of magnitude much larger than the baseline viscoelastic properties of the tube material. Negative values of $\tan\delta$ signify phase angles greater than 90°, and hence a negative value of the composite modulus. This would entail instability of a free unconstrained block; in the present results the Reuss composite element is constrained by the displacement control of the testing machine. The tube ensemble is not of a true Reuss type since the deformation of the tubes is distributed rather than lumped; different regions of the buckled tube contain different amounts of stored strain energy. Moreover, since the buckling process is nonlinear and accompanied by a large change in incremental stiffness, the comparison with the simple linear Reuss model involves the general nature of the singularity rather than the details of the shape of the curves. Even so, the expected singularity indeed appears in the experimental results.

The consequences of non-monotonic force–deformation relations in continuous media have been explored from an applied mathematics perspective (for example James (1979) and Ball (1996)). For example, the range of isotropic elastic constants for strong ellipticity is $G > 0$ and $\nu < 0.5$ or $\nu > 1$ (Knowles and Sternberg 1978), which allows negative Young's moduli. Violation of strong ellipticity in an initially homogeneous solid gives rise to an instability associated with the formation of bands of heterogeneous deformation. These analyses are considered to be applicable to the band structure observed in materials which undergo phase transformations. Indeed, the original Landau (1965) theory assumes an internal energy function which can develop multiple minima at certain temperatures, and hence instability. One does not usually speak of negative stiffness in this regard since, owing to the instability, it is not observed. The present results, by contrast, deal with the effects of constituents of both positive and negative stiffness. A multicell Reuss composite would probably be unstable, however particulate composites with negative stiffness inclusions can be stable (Lakes and Drugan 2000).

Negative stiffness is also known in dynamical systems. The 180° phase shift which occurs above the fundamental natural frequency may be interpreted as a negative stiffness, although it is usually not spoken of in that way. Certain dielectric composites can exhibit extreme behaviour albeit at a very high frequency (Nicorovici et al. 1994). The present concept does not depend on inertial terms: it is not resonant.

Composite systems containing constituents of positive and negative stiffness exhibit giant anomalies in the mechanical damping. They may be called exterlibral since they are on the boundary of balance, or archidynamic since they are based on initial force. Negative stiffness can be achieved in a variety of ways; the use of buckled tubes in the present study is illustrative of the concept. In addition to re-entrant cells (Rosakis et al. 1993) and buckled tubes, one may consider inclusions of materials in the vicinity of phase transitions, by virtue of the unstable equilibrium analysed in the Landau theory (Falk 1980) or systems involving powered fluid flow (Thompson 1982). Tubes need not be macroscopic; they could be nanotubes. Composites with constituents of negative stiffness may be of use in damping layers applied to plates of structural material.

§ 5. Conclusions

The damping $\tan\delta$ of composites with buckled rubber tubes of negative stiffness attains a large peak, orders of magnitude larger than the damping of the rubber, in harmony with expectations of an elementary Reuss model incorporating a constituent of negative stiffness.

References

Ball, J. M., 1996, Nonlinear Mathematics and its Applications, edited by P. J. Aston (Cambridge University Press), pp. 93–119.

Bazant, Z., and Cedolin, L., 1991, Stability of Structures (Oxford University Press).

Christensen, R. M., 1972, Trans. Soc. Rheol., **16**, 603–614.

Falk, F., 1980, Acta metall., **28**, 1773.

Gibiansky, L.V., and Lakes, R. S., 1993, Mech. Mater., **16**, 317.

Gibiansky L. V., and Milton, G. W., 1993, Proc. R. Soc., **440**, 163.

Hashin, Z., and Shtrikman, S., 1963. J. Mech. Phys. Solids, **11**, 127.

James, R. D., 1979, Arch. Rational Mech. Anal., **72**, 99–140.

Knowles, J. K., and Sternberg, E., 1978, J. Elasticity, **8**, 329–379.

Lakes, R. S., 1987, Science, **235**, 1038; 1993, Advd. Mater., **5**, 293–296.

Lakes, R. S., and Drugan, W. J. 2000 (to be published).

Landau, L. D., 1965, Collected Papers of L. D. Landau, edited by D. Ter Taar (New York: Gordon and Breach–Pergamon).

Nicorovici, N. A., McPhedran, R. C., and Milton, G. W., 1994, Phys. Rev. B, **49**, 8479.

Paul, B., 1960, Trans. AIME, **218**, 36.

Rosakis, P., Ruina, A., and Lakes, R. S., 1993, J. Mater. Sci., **28**, 4667.

Thompson, J. M. T., 1982, Nature, **296**, 135–137.