A sensitive piezoelectric composite lattice: experiment

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Abstract

Lattice structures based on bimorph rib elements are fabricated and studied experimentally. The effective piezoelectric sensitivity d is observed to be much larger, by a factor of at least 10,000, in magnitude than that of material comprising the lattice ribs. Bending of the ribs in response to input voltage is responsible for the large sensitivity.

1 Introduction

Piezoelectric materials when stressed mechanically produce an electric polarization and when an electric field is applied, they deform. They are always anisotropic. Specifically [1], under isothermal conditions, the strain ϵ_{ij} depends upon stress σ_{kl} via the elastic compliance J_{ijkl}, upon electric field $\mathcal{E}_k$ in piezoelectric materials with sensitivity modulus tensor d_{kij} at constant temperature. Moreover the electric displacement vector $\mathcal{D}_i$ depends on electric field via K_{ij} which is the dielectric tensor at constant stress and temperature.

$$\epsilon_{ij} = J_{ijkl}\sigma_{kl} + d_{kij}\mathcal{E}_k \tag{1}$$

$$\mathcal{D}_i = d_{ijk}\sigma_{jk} + K_{ij}\mathcal{E}_j \tag{2}$$

If there is a phase angle between pairs of field variables, material properties associated with response to sinusoidal input in time become complex quantities [2] in the context of viscoelasticity (phase δ in the compliance), dielectric relaxation, and piezoelectric relaxation (phase ϕ_{piezo}).

Piezoelectric ceramics have the greatest sensitivity of charge to force, d_{33}, in the reduced notation, 100 to 600 pC/N. This is in comparison with 10 to 20 pC/N for piezoelectric polymers. The sensitivity of displacement to input voltage has the same value, provided the material is elastic; the units pm/volt are equivalent to pC/N.

Piezoelectric composite materials may have a variety of inclusion microstructures including particulate, fibrous, platelet. Piezoelectric composites offer increased electromechanical coupling efficiency, also a decrease in specific acoustic impedance, and suppression of unwanted modes of vibration in comparison with homogeneous piezoelectric ceramic materials. As for two-phase composites with simple geometry, effective piezoelectric coefficients have been calculated [3] and are of rule of mixture form. For composites with more complex geometry, bounds on various physical properties have been developed [4]. Elastic properties of two-phase composites are governed by upper and lower bounds obtained by Hashin [5], and by Hashin and Shtrikman [6]. Thermoelastic properties of two phase composites are bounded [7]. The bounds are weighted averages of the expansion values of the constituents. Bounds for piezoelectric composites [8] [9] are known.

Bounds can be exceeded in several ways. For example, arbitrarily high positive or negative thermal expansions can be achieved in lattice type composites with void content [10] [11] or in dense composites with interfaces that allow slip. Such composites contain rib elements that themselves have composite microstructure, corresponding to a low level of structural hierarchy [12]. Lattices can exceed the limits imposed by bounds that tacitly assume there is no void space and no slip between constituents.

Piezoelectric lattices were designed and analyzed [13]. The effective piezoelectric sensitivity d can be arbitrarily greater in magnitude than that of material in the ribs. Piezoelectric lattices based on two dimensional chiral honeycomb [14] with a negative Poisson's ratio approaching -1 (highly auxetic) are also possible.

A triangular piezoelectric lattice [13] is shown in Figure 1. Rib elements consist of piezoelectric bimorph elements. Such elements undergo bending in response to an electrical signal. Each rib in the lattice consists of two layers of piezoelectric material, arranged as a bimorph. A voltage is applied to such a rib gives rise to piezoelectric deformation in which one layer expands, and the other layer contracts, giving rise to bending. In Figure 1, dark and light shading of the the ribs denotes different polarization directions of layers in the bimorph ribs. Some electrical connections from +, - electrodes are schematically indicated by thin lines.

A summary of the analysis [13] is as follows. Ribs in the lattice in Figure 1, are free to bend unless the lattice is subject to a constraint. The deflection u per volt V for a single bimorph cantilever rib of length L and full thickness h containing two anti-parallel piezoelectric layers of sensitivity d_{31} (in the reduced notation) and thickness $h/2$ is [15] $u/V = -\frac{3}{2}d_{31}[\frac{L}{h}]^2$. The lattice of bimorph ribs contains multiple cells of angle θ between ribs: n cells thick in the z direction, m cells thick in the x direction. The effective sensitivity is:

$$d_{33}^{eff} = -d_{31}\frac{3}{2}\frac{2n}{4}(1 + sin(\frac{\theta}{2}))[\frac{L}{h}]^2. \tag{3}$$

The sign of d_{33}^{eff} depends on the rib orientation. Interchange of layers results in a reversal of sign. This sensitivity is much larger in magnitude than the intrinsic piezoelectric coefficient of the material from which the ribs are made, as a result of the enhancement of displacement by bending. For the lattice shown, $n = 2$ and $\theta = 60°$.

In the present research, we fabricate and test piezoelectric lattices based on composite rib elements. We observe that effective piezoelectric sensitivity d of the lattice can be orders of magnitude greater than that of material in the ribs.

2 Methods

A triangular unit cell was made containing containing three bimorph elements. Lattice ribs were commercial bimorph elements, called stripe actuators by the manufacturer [18]. These had length $L = 48.5$ mm, width $w = 2$ mm and thickness $h = 0.6$ mm, with electrodes near the ends. Further, a lattice of the configuration shown in Figure 1 was assembled. Rib ends were cemented with polymeric glue. Spacers were PMMA plastic. The lattice was oriented vertically. Support was proved by the bottom two spacers; the one on the right was cemented to an optical breadboard and the one on the left consisted of a roller, 6 mm in diameter, to provide vertical support but minimize constraint on the horizontal motion. Electrical input to the unit cell or lattice was via a SRS type DS345 synthesized function generator amplified by an Avtech amplifier. Displacement was measured using an LVDT (Trans Tek, 240-000). This was calibrated using a micrometer driven vertical stage. First the LVDT core was attached in a vertical orientation to a spacer at the top of the lattice to determine vertical motion and infer d_{33}^{eff}. The LVDT core was then attached in a horizontal orientation to the left corner of the lattice to determine horizontal motion and, from low frequency response infer d_{31}^{eff}. Response and phase angle were determined vs. frequency of input signal to determine the sensitivity and intrinsic phase at low frequency as well as the resonant response. The deformation signal and the electrical drive signal were monitored vs. time using a digital oscilloscope (Tektronix TDS 3012). The phase angle between the input voltage and the lattice deformation was measured using a SR 850 lock-in amplifier.

Experiments were also done using square wave excitation. Mechanical and electrical excitation were applied in separate experiments. Mechanical excitation was achieved by providing a second, larger, bimorph cantilever 10 mm wide, 60 mm long, 0.6 mm thick, with its end near the unit cell or lattice. Electrical input to this cantilever was a square wave; sudden input and release of contact force provided the transient input. Square waves revealed the vibration following an impulse and the free decay of vibration associated with mechanical damping. The damping was obtained from transient response from the time $t_{1/e}$ for decay of vibration to a factor $1/e$ of an initial value and the period T of oscillation via $tan\delta = \frac{1}{\pi}\frac{T}{t_{1/e}}$.

Damping may also be enhanced in piezoelectric elements by attaching an electrical circuit that exhibits resonance or has a feedback amplifier [16] or is resistive or resonant [17]. Specifically, an appropriate electrical impedance connected to the circuit provides mechanical damping via the electromechanical coupling that occurs in piezoelectric materials. In the present experiments a resistor matched to the lattice capacitance was used to study such damping. For experiments with mechanical excitation, the resistor was placed across the unit cell or lattice. For experiments with electrical excitation, the resistor was in series with the lattice; the function generator itself has a much lower source impedance, about 50 Ω.

3 Results

The observed effective sensitivity in the z direction for 13 V p-p electrical input at 10 Hz of a single triangular unit cell for displacement between the center of the bottom and the top of the triangle was $d_{33}^{eff} = 2.1$ μm / volt. The effective sensitivity of the lattice was $d_{33}^{eff} = 9.4$ μm / volt. This is far greater than the intrinsic sensitivity of the layers in the ribs, no more than 0.5 nm / volt for the best piezoelectric ceramic that may be used in the ribs. The manufacturer quotes a 30% tolerance for the sensitivity of each bimorph element. There are 30 ribs in the lattice and 8 ribs in a representative deformation path from top to bottom of which four are aligned to give a full sensitivity contribution. The full lattice should be six times as sensitive as a single

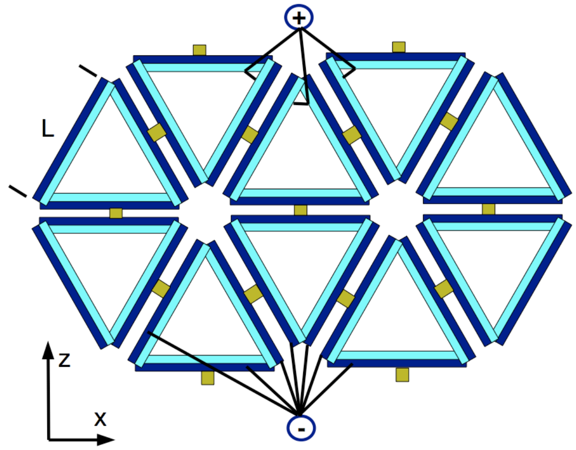

Figure 1: Triangular piezoelectric lattice structure. Small rectangles represent stiff spacers that transmit the bending motion among adjacent cells.

triangle based on Equation 3. The maximum deviation between single cell and lattice sensitivity based on the manufacturer quoted tolerance is a factor of 1.3. Moreover, the lattice is not ideal in that the joints have some rigidity from the glue joints and there may be some friction. The theoretical prediction of large sensitivity of the lattice is supported by experiment. Sensitivity values represent magnitudes. The sign of d sensitivity is understood in the context of electrical contact at the surfaces of homogeneous media or of materials that can be homogenized into an effective continuum. The lattice does not have this kind of electrical connectivity, so a magnitude is reported. As for horizontal sensitivity for input at 5 V p-p and 10 Hz, the effective sensitivity of the lattice was $d_{31}^{eff} = 2.9$ μm / volt. It was necessary to reduce the drive signal to obtain stable response because at high amplitude input the roller moved with time, changing the boundary condition, hence the signal. Linearity was checked by verifying that the response to a sinusoidal input was also sinusoidal.

Response of the lattice in the z direction to sinusoidal electrical input is shown in Figure 2. Response of the lattice in the x direction to sinusoidal electrical input is shown in Figure 3. Resonance is clearly shown in both x and y directions; the fundamental frequency differs because the displacement transducer LVDT core, which has some inertia, was attached at a different location for each measurement. Damping inferred from ratio of the height of the fundamental resonant peak to response at low frequency was tan $\delta = 0.2$ for motion in the z direction and tan $\delta = 0.2$ for motion in the x direction. This is considerably larger than the value obtained for the individual rib. Viscoelasticity of the glue joints is expected to result in increased damping of the lattice. Also there are multiple modes, closely spaced, complicating the interpretation so these damping values are reported to one significant digit.

The mechanical damping of a single rib was inferred from free decay of vibration following a mechanical impulse upon the rib element, Figure 4 (a). The damping was tan $\delta = 0.05$ at the fundamental natural frequency of the cantilever, 139 Hz. Transient response of the triangular unit cell and of the full lattice to an electrical square wave at 10 Hz is shown in Figure 5. As anticipated from the lattice frequency response, considerable damping is manifested in the transient response. For the triangular cell the damping was tan $\delta = 0.18$; for the full lattice, it was about 0.3. The triangular cell and the full lattice clearly have higher damping than a single rib as revealed by transient response in Figures 4 (b) and 5. Because there are multiple modes closely spaced, inference

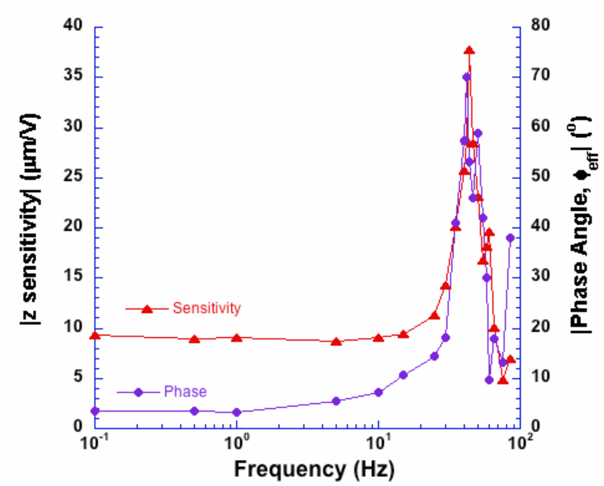

Figure 2: Frequency response of lattice showing resonance. Displacement in the z direction divided by input voltage.

of a quantitative value of damping from the lattice response is problematical. Multiple modes in the full lattice response give rise to a complex time response. The effect of added electrical resistance is to enhance the damping to a value greater than 1 for the triangle cell. For the lattice, it appears that the resistance causes the vibration to damp out more rapidly, but the background damping is already high, and the multi-mode response makes it difficult to precisely quantify damping at resonance. It is notable that electrical excitation favors the fundamental mode while mechanical excitation causes more response at higher modes.

As for piezoelectric phase angles in the response to electrical input, the tangent of phase angle between input voltage and displacement for a single rib as a cantilever was tan $\phi_{piezo} = 0.03$ at a frequency 1 Hz and 10 volts p-p. This is the ratio of the imaginary part to the real part of d_{33}^{eff} for the bimorph rib. This is conceptually different from the mechanical damping. For the full lattice, Figure 2, at low frequency 1 Hz, well below resonance, the piezoelectric phase between voltage and displacement waveforms was tan $\phi_{piezo} = 0.07$ in the z direction and tan $\phi_{piezo} = 0.04$ in the x direction. It appears that viscoelastic damping in the joints in the lattice contributes to piezoelectric phase angle.

4 Discussion

The present piezoelectric lattice was assembled from commercial bimorph elements. For macroscopic or microscopic lattices, one may also use prototype methods previously used for composites [19]. Polymeric lattices [20] called micro-truss structures have been made by using controlled beams of light to polymerize a precursor liquid containing monomer; they are not piezoelectric. Lattices can be made with any cell size from macroscopic to nano-scale. There is no length scale in the classical theory of elasticity or piezoelectricity. The piezoelectric lattice has effective properties but it should not be viewed as an equivalent continuum. The reason is that this piezoelectric lattice receives electrical input via connectivity to appropriate ribs within the lattice structure rather than at the surface as is the case for homogeneous materials. That gives rise to a dependence of effective sensitivity on the number of cells in a particular direction. Such behavior is similar to that of stacks of axial piezoelectric elements; these have electrical connectivity such that a low voltage applied

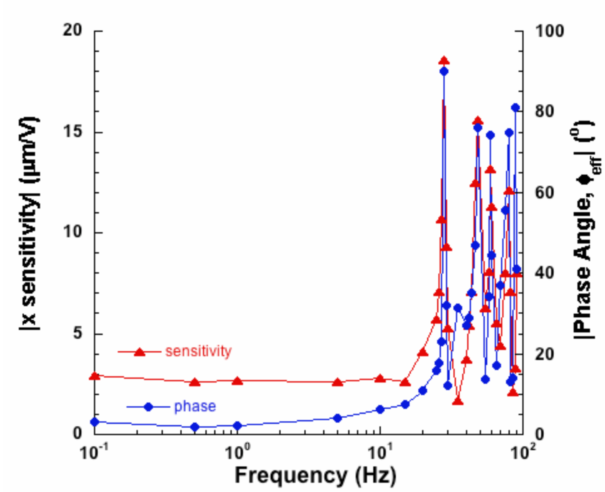

Figure 3: Frequency response of lattice showing resonance. Displacement in the x direction divided by input voltage.

results in a sum of displacements of each element.

The piezoelectric sensitivity of this lattice is much greater than that of the rib constituents. Such sensitivity exceeds bounds for two-phase composites. Mathematical bounds for two phase composites do not apply to piezoelectric lattices as in the case of thermoelastic lattices that exhibit thermal expansion. The reason is the bound analysis tacitly assumes the two phases are perfectly attached together with no slip or void space and that they are in a minimum energy state [21]. The no void assumption does not apply to lattice structures considered here. It is possible to envisage three phase bounds; the void space would be considered as a third phase as incorporated [11] in the interpretation of lattices of high thermal expansion.

Phase angles at low frequency well below resonance occur in mechanical, piezoelectric and dielectric properties. The tangent of the phase angle between stress and strain, tan δ, is a measure of mechanical damping. Piezoelectric materials exhibit viscoelastic response associated with the material itself. Bimorph elements may exhibit additional damping associated with interfaces. In piezoelectric materials, the electrical boundary conditions including effects of shape, influence the mechanical damping [22]. Damping in the lattice is greater than that of a single rib. This is attributed to damping elsewhere in the structure, including glue joints between ribs. Damping was enhanced further by the addition of an electrical resistance.

Piezoelectric lattices may be used in applications such as actuators, energy harvesting, and structures that change shape in response to stimuli. Piezoelectric lattices are, however, too compliant to be used as ultrasonic resonators. Three-dimensional lattices may be envisaged based on similar principles, for example an array of cells in the form of cubes or other polyhedra with bimorph ribs, connected to each other at the rib midpoints.

5 Conclusion

Lattice structures based on bimorph rib elements made and measured in the laboratory exhibit effective piezoelectric sensitivity $d_{33}^{eff} = 9.4$ μm / volt. This is much larger, by a factor of at least 10,000, in magnitude than that of material comprising the lattice ribs. Bending of the ribs in

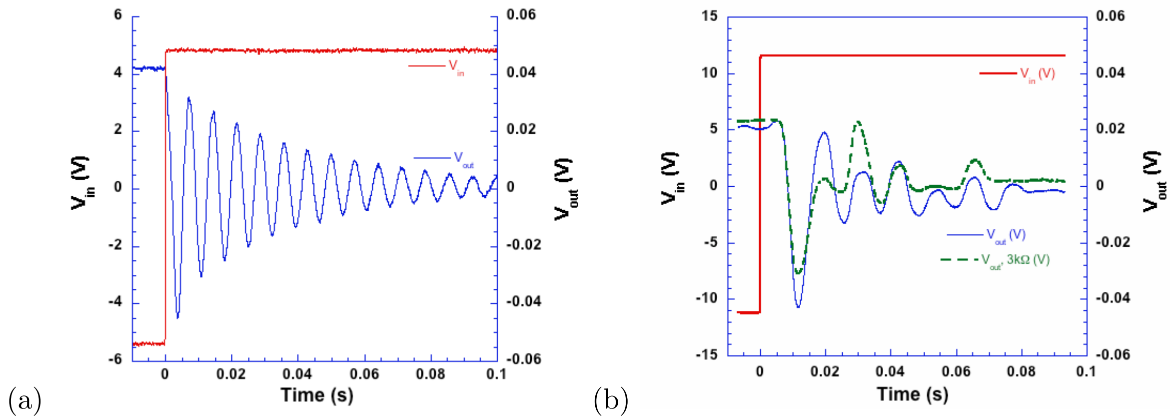

Figure 4: Transient response to a mechanical impulse: (a) a single rib as a cantilever, (b) full lattice in z direction, with no attached resistance and with 3 kΩ resistance.

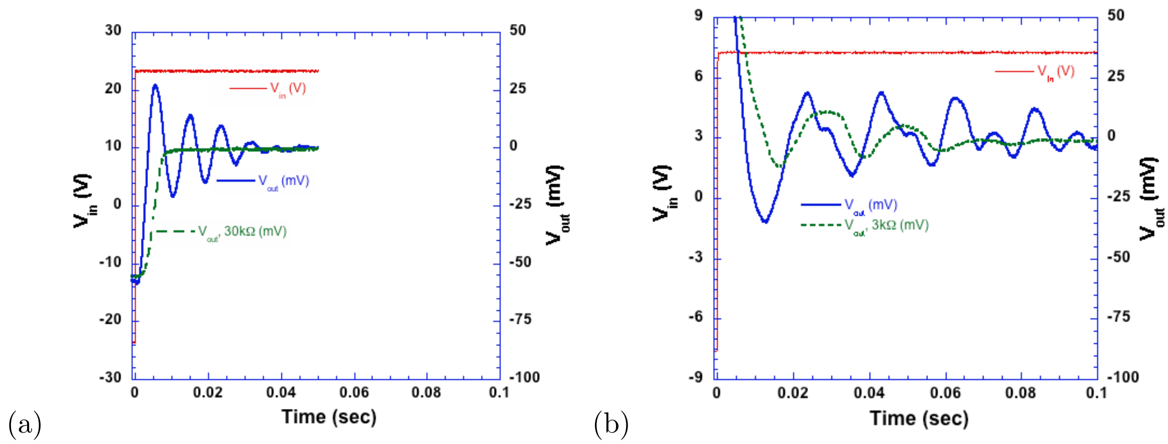

Figure 5: Square wave response in z direction, electrical excitation; (a) triangle cell with no attached resistance; triangle cell with 30 kΩ resistance; (b) full lattice, with no attached resistance and with 3 kΩ resistance.

response to input voltage is responsible for the large sensitivity.

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