A unit cell structure with tunable Poisson's ratio from positive to negative

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Abstract:

A novel unit cell structure with re-entrant hollow skeleton is designed and its Poisson's ratio has been studied using the finite element method (FEM) as a function of the geometric variables and the parent material Poisson's ratio. The simulation results showed that the Poisson's ratio of the unit cell structure is tunable over the full range of Poisson's ratio (-1 to +0.5) by varying the geometric variables and parent material Poisson's ratio. Three types of samples with identical geometric variables were fabricated via 3D printing using three different materials, and their Poisson's ratios were measured and compared with the ones from FE simulation, and excellent agreement was found between the experimental results and FE simulation.

Keywords: Re-entrant cellular structure; Negative Poisson's ratio; Simulation and modeling; Elastic properties; Structural;

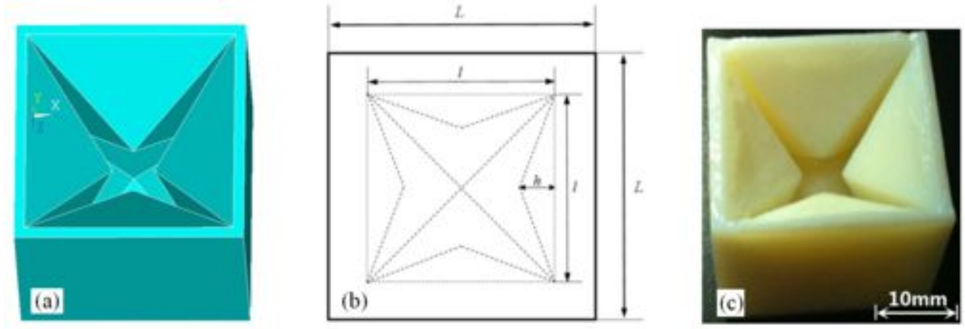
1. INTRODUCTION

Poisson's ratio (ν) is defined as the ratio of the transverse contraction strain to the longitudinal extension strain in tension. Poisson's ratio can range from -1 to 0.5 for isotropic materials, but most solids have a positive Poisson's ratio (0.25-0.33). Negative Poisson's ratio was initially studied in 2-D structures[1-3]. Negative Poisson's ratio in 3-D isotropic materials has been observed in designed cellular materials with re-entrant structures[4,5], polymer gels[6] and ferroelastic ceramic [7] near phase transition temperatures and in In-Sn alloy [8, 9] near a morphotropic phase boundary. Negative Poisson's ratio is also known in anisotropic materials such as orthorhombic alloy [10], and certain in-plane directions of 3D structures [11, 12] and in more recent study of 2-D structures[13-15].

In this paper, a unit cell structure with a re-entrant hollow skeleton has been designed, analyzed, fabricated and tested. The unit cell Poisson's ratio has been studied using FEM to investigate the effects of geometric variables and parent material properties upon the unit cell Poisson's ratio. Three samples with identical geometric variables were fabricated by 3D printing from three different materials (Vero White Plus, Tango Black Plus and PLA), and their Poisson's ratios were measured and compared with simulation results; excellent agreement was found between measurements and simulation. This design concept can be used to fabricate hybrid cellular structures with diverse physical and mechanical properties at different parts of the structures to meet special robust mechanical performance requirements.

2. SIMULATION

The Poisson's ratio of a unit cell structure with a re-entrant hollow skeleton has been studied using the commercial finite element software ANSYS (see Fig. 1a, b). Poisson's ratio of the parent material used to make the structure was set to vary from 0 to 0.5 with an incremental step of 0.1. Free meshes of 82980 elements (Solid 186, 20 nodes) were used. h/l was set to vary from 0 to 0.45 with an incremental step of 0.025, and L/l was set to vary from 1.1 to 1.4 with an incremental step of 0.1. The bottom surface of the unit cell model is fixed, with all degrees of freedom being constrained, and the compressive strain was set as 0.1% on the top surface of the model in the vertical direction. It was found that the simulation results of the unit cell Poisson’s ratio are independent of the Young’s modulus of the parent material.



3. EXPERIMENT

In this paper, Object Eden 260 3D Printer and MakerBot Replicator z18 3D Printer were used to fabricate the unit cell samples. Three types of unit cell samples with identical geometric variables were printed from three different materials: Vero White Plus, Tango Black Plus, and PLA. Vero White Plus has the highest stiffness among the three materials while the Tango Black Plus is most compliant. A printed sample (made from Vero White Plus) is shown in Fig. 1c. One face of the cube was removed to show in the inner structure. The geometric variables of the printed samples are h=12mm, l=30mm, L=33mm.

Cubic materials have equal moduli and Poisson's ratios in the principal directions but can differ in oblique directions. Cubic materials have three independent elastic constants [16]. The unit cell samples were compressively loaded in the Z direction using a universal testing machine (Zwick010, Germany) with a testing speed of 2mm/min (corresponding to a strain rate of 10^{-3}/s). The transverse displacements in both the X and Y directions in the middle plane of the unit cell sample were recorded by a digital outside micrometer (Guilin Measuring and Cutting Tool Co. Ltd. China) at the unit cell 0.4% normal strain for Vero White Plus and 2% normal strain for Tango Black Plus and PLA. The strains were selected because they are lower than the unit cell elastic limit but sufficiently high that the end effect (due to frictions between the test sample and the surfaces of the platens) can be neglected. The high machining precision of 3D printing route ensures a geometrically accurate final product, and negligible difference was found between the displacement measurements in the X and Y directions; the arithmetic mean of the X and Y direction displacements was recorded as the transverse strain. The arithmetic mean of the transverse strains recorded from 8 separate measurements was used to determine the unit cell Poisson’s ratio v. If the symmetry is orthotropic, there are three independent Poisson's ratios. To test symmetry, load was applied in each principal direction and Poisson's ratio measured. Negligible difference (less than 5%) was found between ν_{xy}, ν_{yz}, and ν_{xz}, a fact that suggests good cubic responses of the fabricated unit cell structure. In this study, only ν_{xy} is reported.

4. RESULTS AND DISCUSSION

4.1. Simulation results

Poisson's ratio is defined as the ratio of transverse contraction strain to longitudinal extension strain in the direction of stretching force as:

$$\nu = -\frac{\varepsilon'}{\varepsilon} \qquad (1)$$

Where, ν is the Poisson's ratio, ε' and ε are the transverse contraction strain and longitudinal extension strain, respectively.

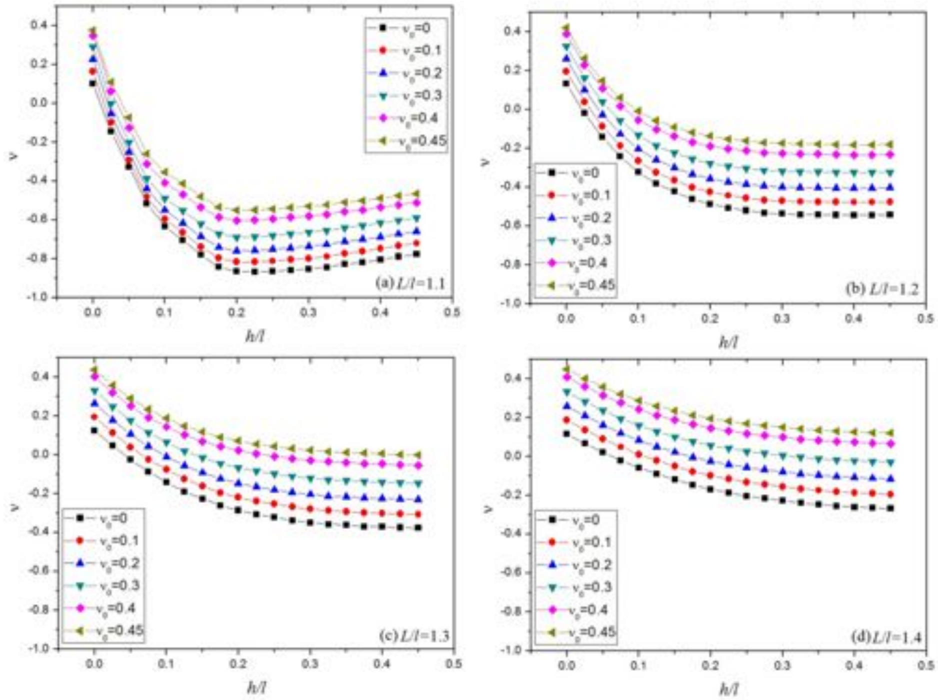
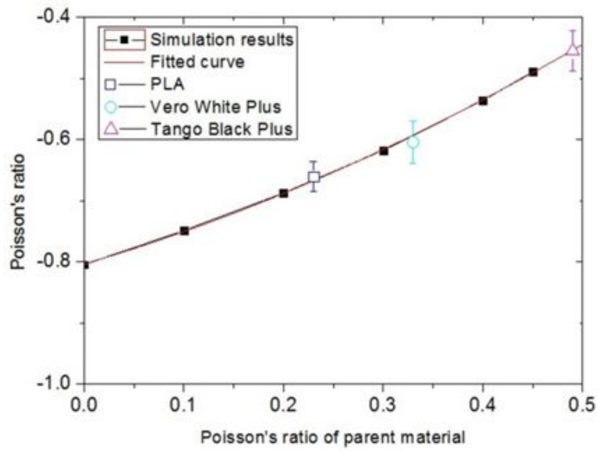


Fig. 2 shows the relationships between the simulated Poisson's ratio of the designed unit cell and its *h/l* ratio from 0 to 0.45 when the *L/l* ratio is varied from 1.1 to 1.4. The unit cell Poisson's ratio decreases with a decreasing value of the Poisson's ratio (ν_0) of the parent material used to make the structure. The unit cell Poisson's ratio exhibited a similar behavior for different parent material Poisson's ratios (ν_0): for any specific Poisson's ratio of the parent material, the unit cell Poisson's ratio decreases rapidly with an increasing *h*/*l* ratio from 0 to 0.2; the unit cell Poisson's ratio decreases at a much slower rate beyond *h/l*= 0.2 in all cases except *L/l*=1.1, where a slight increase is observed after *h/l*=0.2. The simulation results have shown that the unit cell Poisson's ratio is tunable from positive to negative with an increasing *h*/*l* and an appropriate combination of *L/l* and the parent material Poisson's ratio. A minimum unit cell Poisson's ratio, -0.84, is achieved when *h*/*l*=0.2, *L*/*l*=1.1 and ν_0=0. The simulation results show the full range of the Poisson's ratio as a function of geometric variables; the wide tunable range of the Poisson's ratio is of interest for practical designs. Classical elasticity has no length scale, so there are no size effects in classical elastic moduli. The single cell will be equivalent to a lattice made of such cells provided the links between cells replicate the boundary conditions assumed and provided applied stress is uniform. In future lattices, cells can be made smaller and lattices can be made with many cells. It is possible that by stacking unit cells with specially designed geometric parameters hybrid cellular materials with desired mechanical properties can be synthesized. A cellular material constructed by stacking identical unit cells is expected to exhibit homogeneous constitutive parameters; however, the properties of a material constructed by stacking hybrid unit cells requires further study.

4.2. Experimental results

Three types of unit cell samples with identical geometric variables were manufactured by 3D printing from three different parent materials: Vero White Plus, Tango Black Plus, and PLA, the Poisson's ratios of which are 0.33, 0.49, and 0.23 respectively. Two samples were made for each type of unit cell designed in the present study. The measured Poisson's ratios of the total six unit cell samples were plotted against their parent material Poisson's ratios and compared with the simulation results (see Fig. 3). The measured unit cell Poisson's ratios were -0.455±0.033 (standard deviation), -0.604±0.035 and -0.660±0.024 for samples made from Tango Black Plus, Vero White Plus, and PLA, respectively.



The least square method was used to curve fit the simulation results (solid squares in Fig. 3):

$$\nu=0.47\nu_0^2+0.48\nu_0-0.80 \quad (2)$$

In Equation (2), ν refers to the unit cell Poisson's ratio and ν_0 refers to the parent material Poisson's ratio. It can be seen that the measurements are in excellent agreement with the simulation results.

5. CONCLUSIONS

A novel unit cell structure with a re-entrant hollow skeleton has been proposed and studied via FEM. The unit cell Poisson's ratio is found to be tunable between -1 and 0.5 by varying the geometric variables and the parent material Poisson's ratio. Three types of unit cell samples with identical geometric variables were fabricated via 3D printing from three different materials, and the unit cell Poisson's ratios were measured during uniaxial compression testing. The measurements have showed excellent agreement with the simulation results. The design concept proposed in the present study can be used to fabricate hybrid cellular lattice structures with diverse physical and mechanical properties at different parts of the structures. As structures with negative Poisson’s ratio have higher indentation resistance and higher energy absorption capacity than conventional materials, the hybrid cellular materials constructed by specially designed 3D packing of the proposed unit cell structures with diverse geometric and material variables could have potential applications in such as sports facilities, aerospace fillers and bio-implants.

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References

[1] Gibson L J, Ashby M F, Schajer G S, Robertson C I, Proc. R. Soc. Lond. A, 382 (1982).25-42.
[2] Kolpakov A G, Prikl. Mat. Mekh 49 (1985) 969-977.
[3] Wojciechowski K W, Physics Letters A, 137 (1989) 60-64.
[4] Lakes R S, Science, 238 (1987) 551.
[5] Buckmann T, Schittny R, Thiel M, Kadic M, Milton G W, Wegener M, New J Phys, 16 (2014) 033032.
[6] Hirotsu S, J. Chem. Phys. 94 (1991) 3949.
[7] Dong L, Stone D S, R.S. Lakes, Philos. Mag. Letts. 90 (2010) 23-33.
[8] Li D, Jaglinski T M, Stone D S. Lakes R S, Appl. Phys. Lett. 101(2012) 251903.
[9] Li D, Dong L, Lakes R S. Materials Letters, 143 (2015) 31-34.
[10] M. Rovati, Scripta. Mater. 48 (2003) 235-240.
[11] Hu H, Silberschmidt V, J Mater Sci, 48 (2013) 8493-8500.

[12] Hou X N, Hu H, Silberschmidt V. Composites: Part B, 58 (2014) 152-159.
[13] Prawoto Y, Comput Mater Sci, (2012)140-153.
[14] Kaminakis N T, Stavroulakis G E, Composites: Part B, 43(2012)2655-68.
[15] Pozniak A A, Smardzewski J, Wojciechowski K W. Smart Mater Struct, 22 (2013) 1-11.
[16] Nye, J. F., Physical Properties of Crystals, Oxford, Clarendon, 1976.

Figure caption

Fig. 1 (a) The ANSYS model of the unit cell structure with a re-entrant hollow skeleton; (b) schematic drawing of the unit cell structure viewed in the x-z plane; (c) a sample fabricated by 3D printing from Vero White Plus.

Fig. 2 The relationships between the Poisson's ratio of the designed cellular structure and the *h/l* ratio from 0 to 0.45. Plot (a), *L/l* = 1.1; (b), 1.2; (c), 1.3; (d), 1.4. Symbols (■), (□), (▲), (▼), ()and(◄)-represent different parent materials with Poisson's ratio of 0, 0.1, 0.2, 0.3, 0.4, and 0.45, respectively, used to make the designed cellular structures.

Fig. 3. Poisson's ratio *v* of the cellular structure as a function of that of the parent material used to fabricate the structure. The experimental data of samples made from PLA, Vero White Plus and Tango Black Plus are represented by (), (), and (), respectively. (-■-) represents the simulation results, and () is the fitted curve for the simulation results.