Dynamic wave dispersion and loss properties of conventional and negative Poisson's ratio polymeric cellular materials

by

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This article describes experimental investigations of the dynamical behaviour of conventional and negative Poisson's ratio foamed materials in torsional vibration. Dispersion of standing waves and cut-off frequencies were observed. Consequently, foamed materials do not obey the classical theory of elasticity or viscoelasticity. The dynamical effects were attributed to micro-vibrations of the cell ribs in a structural view and were associated with microstructure or micromorphic elasticity in a continuum view. Cut-off frequencies were lower in re-entrant foams with negative Poisson's ratios than in the conventional foams from which they were derived. An analytical structural model was developed in which the ribs of the conventional foams were modeled as free-free vibrating beams. The predicted cut-off frequencies were comparable to those observed experimentally.

1 INTRODUCTION

Conventional foams like other ordinary materials, foams exhibit a positive Poisson's ratio, that is, they become smaller in cross-section when stretched and larger when compressed. Recently, the invention of negative Poisson's ratio foams was reported[1,2,3]. These foams exhibit negative Poisson's ratio as small as -0.7 as well as enhanced resilience.

Cellular solids, although they contain visible structure, are often modeled as continuous media for engineering purposes. For example, in the classical theory of elasticity, the actual material is replaced by an equivalent continuum in which points have translational degrees of freedom only, and the transmission of load across a differential element of surface is described completely by a force vector. The predictions of elasticity theory agree with experiment for most engineering materials under most circumstances. For structured materials such as composite materials and foamed materials, if the physical dimensions exceed the microstructure size by several orders of magnitude, classical elasticity is considered to be adequate. If not, a more general continuum model may be needed. For example, the Cosserat (micropolar) model incorporates local rotation of points, which corresponds to rotation of structural elements, and couple stress, which represents an average of the bending and twisting moments upon structural elements. The Cosserat model can account for many static effects (size effects and changes in stress concentration) and dynamic effects (dispersion of shear waves and the existence of new types of waves) in structured materials. The Cosserat continuum model has been found to describe low density[4] and high density[5] polymeric foams in bending and torsion more accurately than the classical continuum model. All six of the Cosserat elasticity constants were found[5]. In a related vein, theoretical evidence of unusual wave behaviour in gridworks has been predicted as well[6]. Dispersion of waves in solids has been explored experimentally in foams, viscoelastic materials, and composite materials[4,7,8,9].

It is the purpose of this study to investigate the wave dispersion of conventional and negative Poisson's ratio foamed materials. The dynamic Young's modulus and viscoelastic loss were obtained from experimental data as well. Two experimental methods used for this study are: a free-free resonance experiment in which standing waves are excited in the specimen at a limited number of discrete frequencies, and a micromechanics apparatus in which the dynamic compliance and phase are measured over a wide range of frequency.

2 MATERIALS AND METHODS

Free-free resonance experiments in torsion were conducted at room temperature. Two types of viscoelastic foams, polyester foam and Scott industrial foam, were tested. The polyester foam used here is partly open cell and partly closed cell with cell size 0.5 mm and density 0.031g/cm^3; Scott industrial foam is all open cell with cell size 2.5 mm and density 0.0304g/cm^3. For comparative purpose, re-entrant foams of negative Poisson's ratio were tested as well as conventional foams. Specimens of round and square cross section 12 mm to 25 mm across were used in the free-free resonance experiments, and round specimens 12.7 mm in diameter were used in the micromechanics apparatus.

2.1 Free-free resonance experiment

The apparatus and method are similar to those used earlier by various authors for study of micromechanics of polymeric foams[4] and hard viscoelastic solids[10]. The apparatus is as shown in Figure 1. Torsional vibrations are excited in the foam by an electromagnetic technique in which an oscillatory current passed through the drive coil attached to one end of the specimen and immersed in a static magnetic field. A gated sinusoidal electric current of known frequency in generated, amplified, and injected into the drive coil to produce a torque upon the specimen. The specimen vibration is detected using a similar coil at the same end of the specimen. High intensity samarium cobalt disc magnets, 25 mm in diameter and 10 mm in thickness, are used to generate the largest possible magnetic field. The coils are made from the finest available wire to reduce the inertia added to the specimen. The wire is 0.1 mm in diameter, of gage number 38, and 6.7*10^{-5} g/mm in linear density. The excitation waveform is a series of gated bursts of sinusoids. The detected free-decay waveforms are displayed and recorded on the digital oscilloscope. Resonant frequencies thus determined as the frequency associated with a phase of 90° between the detected waveforms and the drive signal; this corresponds approximately to the peak in vibration amplitude. In addition to the fundamental resonance, the second and third resonances are also detectable up to approximately 500 Hz if the specimen is sufficiently long.

In initial tests, the specimen was supported horizontally with an extra wire or a probe. It was found that the resonant frequency was significantly affected by the presence of the wire and the probe. It was therefore decided to adopt the experimental configuration as shown in Figure 1.

Data reduction is based on the following analytical considerations. The classical theory of viscoelasticity predicts the dynamic rigidity for free-free torsional vibration to be[11]

$$M^*/\Phi = I_{sp}\omega^2 [\tan\Omega^*/\Omega^*] - I_{at}\omega^2 \qquad (1)$$

in which M* is the sinusoidal torque applied, Φ is the end angular displacement, I_{sp} is the mass moment of inertia of the specimen, ω is the angular frequency in radians per second, $\Omega^* = [\rho\omega^2 h^2/KG^*]^{1/2}$, ρ is the mass density of the specimen, h is the length of the specimen, K is a geometrical constant, 1 for a specimen with circular cross section; 0.8 for a specimen with square cross section, G* is the complex shear modulus, and I_{at} is the mass moment of inertia of the end attachment.

It is suggested by Equation (1) that the fundamental resonance occur at $\tan\Omega^* = \Omega^* \, I_{at}/I_{sp}$ and at $\Omega^* = \pi$ if $I_{at}=0$. $\tan\Omega^*$ can be approximated to be $\Omega^* - \pi$ by expanding Ω^* about π since $\Omega^* \quad \pi$ if $I_{sp} >> I_{at}$. The fundamental resonant frequency of the specimen ν_{sp} (in Hz) is therefore determined to be

$$\nu_{sp} \quad (1 + I_{at}/I_{sp})\, \nu_{exp} \qquad (2)$$

in which ν_{exp} (in Hz) is the observed fundamental resonant frequency, which depends on the total inertia of the specimen and the attachment. Resonant frequency always refers to ν_{sp} in the later sections unless otherwise indicated. Equation (2) was considered appropriate to derive ν_{sp} from ν_{exp} since I_{at}/I_{sp} was at most 15% in the experiments.

Observations from free-free experiments can be used to determine viscoelastic properties of the specimen as well. The absolute value of the shear modulus |G*| is derived from the approximation $\Omega^* = 2\pi\, \nu_{sp} h[\rho/KG^*]^{1/2} \quad \pi$ at resonance, and is given by

$$|G^*| = \rho\, (2\, \nu_{sp} h)^2 / K \qquad (3)$$

Based on the fact that resonance occurs when the specimen length h is equal to one half of the wavelength, |G*| can also be determined as[11]

$$|G^*| = \rho\, (2\, \nu_{sp} h \cos(\delta/2))^2 / K \qquad (4)$$

However, the approximation error in |G*| obtained from the simpler Equation (3) is smaller than the thickness of plotting data points for small loss materials. 2% error in |G*| occurs for a typical loss tangent tan δ of 0.3 for polyester foams.

Tan δ at the resonant frequency is determined from two consecutive amplitudes A_n and A_{n+1} of free-decay waveforms by[10]

$$\tan\delta = \ln(A_n/A_{n+1})/\pi \qquad (5)$$

2.2 Micromechanics apparatus experiment

The experimental method and the associated data reduction for the micromechanics apparatus experiment was given in detail in earlier publications[12,13]. An experimental apparatus and analysis scheme were developed for determining the viscoelastic properties of a material isothermally, with a single apparatus, over 10 decades of time and frequency. Torque was applied to the specimen electromagnetically and its deformation was determined by laser interferometry. Resonances were eliminated from the torque and angle measuring devices by this approach. Shape resonances remaining in the specimen itself were corrected by a numerical analysis scheme based on an analytical solution which is applicable to homogeneous cylindrical specimens of any degree of loss. The apparatus is capable of creep, constant load rate, subresonant dynamic, and resonant dynamic experiments in bending and torsion. The range of equivalent frequency for torsion is from less than 10^{-6} Hz to about 10^4 Hz. This range is superior to the one or two discrete frequencies available in free-free resonance. However, since the analysis scheme for the micromechanics apparatus incorporates the assumption of a continuous viscoelastic medium, interpretation of results in the case of foams is less straightforward than in the case of free-free resonance.

2.3 Microstructural analysis

The foam micro-resonance behaviour can be analyzed by considering the foam rib as a vibrating beam. The cut-off frequency is derived from the relation for the lowest mode of free-free transverse beam vibration[10] and from a relation extracted from the theory of elasticity of open-cell foams[14]. The relation for beam vibration is, $E_s'=12.54\rho_s(L^4/R^2)\omega_o^2$, in which E_s' and ρ_s are the storage component of the Young's modulus and mass density of the solid beam, L is its length, R is the radius of the cross section of the beam, and ω_o is the resonant frequency of the beam in radians per second. For an open cell foam, $E_s/\rho_s=(E/\rho)(h_{rib}^2/4R^2)$, in which E_s and ρ_s are the Young's modulus and mass density of the solid of which the foam is made, E and ρ are the Young's modulus and mass density of the foam, and h_{rib} and R are the length and radius of the foam rib. We let the beam length be equal to the length of the ribs in the foam. From the above, the rib resonance frequency ν_{rib} (in Hz), hence the cut-off frequency, of open cell foamed materials is obtained as

$$\nu_{rib} \quad 0.0225\,(E/\rho)^{1/2} / h_{rib} \qquad (6)$$

in which E and ρ are the Young's modulus and mass density of the foam, and h_{rib} is the rib length. The shear modulus G was measured, so E was calculated from $E = 2G(1 + \nu)$ in which the Poisson's ratio ν for conventional foams is ≈0.3. The rib length was found to vary from cell to cell and within cells but was about 60% to 80% of the cell size. Substitution of measured densities, rib lengths, and Young's moduli of the present conventional polymeric foams in Equation (6) yields predicted cut-off frequencies for these foams. Cut-off frequencies are predicted to be 2 kHz and 550Hz for polyester foam and Scott industrial foam respectively.

Wave dispersion in the foamed materials may be also examined[4] in light of the continuum theory of elastic materials with microstructure[15], which is more general than the Cosserat micromechanics theory referred to above. Such materials are also known as micromorphic materials. A dimensionless technical constant   may be defined as follows:

$$^2 \equiv (2\pi\,\nu_{cut})^2 \rho\, d^2_{cell} / 3G \qquad (7)$$

in which d_{cell} is the cell size and ν_{cut} is the cut-off frequency associated with microstructural micro-vibration in the specimen material. A classically elastic material does not exhibit any micro-vibration resonance, so that ν_{cut} is infinitely large, as is . Small values of   correspond to dynamically 'floppy' cells and to a low cut-off frequency in relation to stiffness and density. Values of   of foam specimens based on experimentally obtained ρ, ν_{cut} and $|G^*|$ are presented in the following.

3 RESULTS AND DISCUSSION

Figures 2-6 contain results obtained by free-free resonance experiments. Dispersion curves for standing waves (resonant frequency vs inverse length) are shown in Figures 2-4. Figure 2 shows results for normal and re-entrant small cell polyester foams; Figure 3 shows results for normal and re-entrant Scott industrial foams; and Figure 4 shows a synopsis of results for both kinds of conventional foam.

These results are interpreted as follows. The slope of the dispersion curve is proportional to the group velocity of waves in the material: $v_g = d\omega/dk$ in which $\omega = 2\pi\nu$ with ν as the wave frequency and $k = 2\pi/\lambda$ with λ as the wavelength, with λ equal to twice the specimen length for free-free resonance. Consequently Figures 2-4 indicate that these foam materials exhibit dispersion of waves (that is, dependence of wave speed on frequency), and cut-off frequencies, at which the slope, hence the group velocity drops to zero.

Dispersion can result from (i) viscoelastic loss, (ii) structural effects modelled by Cosserat elasticity, (iii) structural effects resulting from micro-vibration of the structural elements in the material. Both viscoelastic loss and Cosserat elastic effects are manifested as an increase in wavespeed with frequency, hence a dispersion curve which is concave up. Such behaviour occurs at the lower frequencies as shown in Figures 2-4. The opposite dispersion and cut-off effect seen at the higher frequencies cannot result from viscoelastic or Cosserat behaviour but it can result from micro-vibration. Such effects are known at MHz frequency in composite materials [9] and at $\approx 10^{11}$ Hz in atomic lattices. For the purpose of comparison with the microstructural model, a summary of specimen dimensions, cut-off frequencies, |G*| at a representative frequency, and wave dispersion constant   derived from Equation (7) is presented in Table 1. The column for compression ratio refers to the permanent volumetric compression used to create the re-entrant foams with negative Poisson's ratio[1,2,3]. The cut-off frequencies of conventional foams 2500Hz for polyester foam and 1000Hz for Scott industrial foam are higher than the corresponding 2000Hz and 550Hz predicted from Equation (6). The model is, however, idealized in that free-free vibration may not correspond to the true boundary condition of the ribs in the foam. Moreover, the foam may contain membrane and plate elements as well as rib elements, and these will have different resonant frequencies. Within these limitations, agreement is considered reasonable. A continuum view can also be taken in the analysis of micro-vibrations and cut-off frequencies[15]. The theory is more complex and more general than Cosserat elasticity, allowing 18 elastic constants for an isotropic material, as opposed to six for a Cosserat solid, and two for a classical one. The theoretical dispersion curves are similar to the experimental ones reported here, however a full determination of all of the 18 elastic constants would require many additional experiments. Values of   for conventional foams are somewhat higher than those for the corresponding re-entrant foams. This agrees with the prediction that the re-entrant foams are more affected by the micro-vibration than the conventional foams.

Comparisons of the conventional foams are shown in Figure 4. Polyester small cell foams were found to be stiffer than Scott industrial foams, based on the initial slope of the dispersion curves in Figure 4. The observed cut-off frequencies are in a ratio of 2.5:1. Prediction on the basis of Equation (6) gives a ratio of about 8:1. However there is the following structural difference. Polyester small cell foams are partly closed cell and Scott industrial foams are 'reticulated' and are all open cell. Partly closed cells have a thin membrane cell wall which might resonate at a lower frequency than the ribs of all open cell foams.

Re-entrant foams were found to have lower resonant frequencies for equivalent length as well as lower cut-off frequencies compared with conventional foams. The lower resonant frequencies are a consequence of higher densities and lower stiffness of re-entrant foams than those of conventional foams[3]. Reduction in the cut-off frequency for re-entrant foam is considered to be a result of its higher density, lower modulus, and more convoluted cell structure. The specimen size dependence shown in Figure 2 is considered to be a result of a surface effect in which incomplete cells at the surface contribute less to the rigidity than complete interior cells.

Viscoelastic properties derived at frequencies well below the cut-off frequency, for which the viscoelastic continuum view is appropriate, are shown in Table 1 and Figures 5 and 6. The loss tangent tan δ of the re-entrant foam was approximately consistent within error bars with those of conventional foams as shown in Figures 5 and 6, indicating the transformation process did not grossly change the mobility of the molecules in the polymers. Moreover, the loss tangents of the two types of foam were similar. The larger error bars at the higher frequencies were a result of weaker signals at these frequencies.

The above results were obtained from free-free resonance in which specimens were cut progressively shorter to change the frequency. Experiments were also conducted with a micromechanicsapparatus[13] capable of determining viscoelastic properties over a wide range of frequency upon a single specimen. Normalized shear modulus |G*|/Go and loss tangent tan δ of conventional polyester foams obtained from this apparatus are shown in Figures 7 and 8. Values of the shear modulus |G*| at a frequency of 0.1 Hz were denoted as Go and presented in Table 2. It was notable that both |G*|/Go and tan δ as shown in Figures 7 and 8 disclose unusual behaviour near 1KHz. However, the analysis scheme corrected for specimen shape resonance was under the assumption of a continuous viscoelastic medium[13] and not so straightforward as in the case of free-free vibration. Nevertheless, the effect of specimen length and unusual behaviour near 1KHz in Figures 7 and 8 was not seen in homogeneous materials[12,13], and can be construed as indirect evidence of the dispersive effects referred to above. The loss tangent of conventional foams obtained from free-free torsion resonance experiments and micromechanics apparatus experiments are compared on the same linear scale in Figure 9. Agreement is reasonable below 500 Hz.

Cellular solids exhibit many useful characteristics not found in the homogeneous solids from which they are derived[16]. Applications of conventional foams in packaging, low density cores, filters, and insulation depend upon the quasistatic properties of these materials. The present work reveals unusual dynamical behaviour of both conventional and negative Poisson's ratio foams. The cut-off frequency and the associated vanishing of the wave group velocity may give rise to applications in vibration and sound isolation.

4 CONCLUSIONS

1. Dispersion of standing waves and cut-off frequency effects were observed in polymer foam materials. Cut-off frequencies of conventional foams obtained from the experimental method were somewhat higher than those predicted based on an elementary model of the foam rib vibration.
2. These effects were attributed to micro-vibrations of the cell ribs in the foams.
3. Cut-off frequencies were significantly lower in re-entrant foams with negative Poisson's ratios than in the conventional foams from which they were derived.

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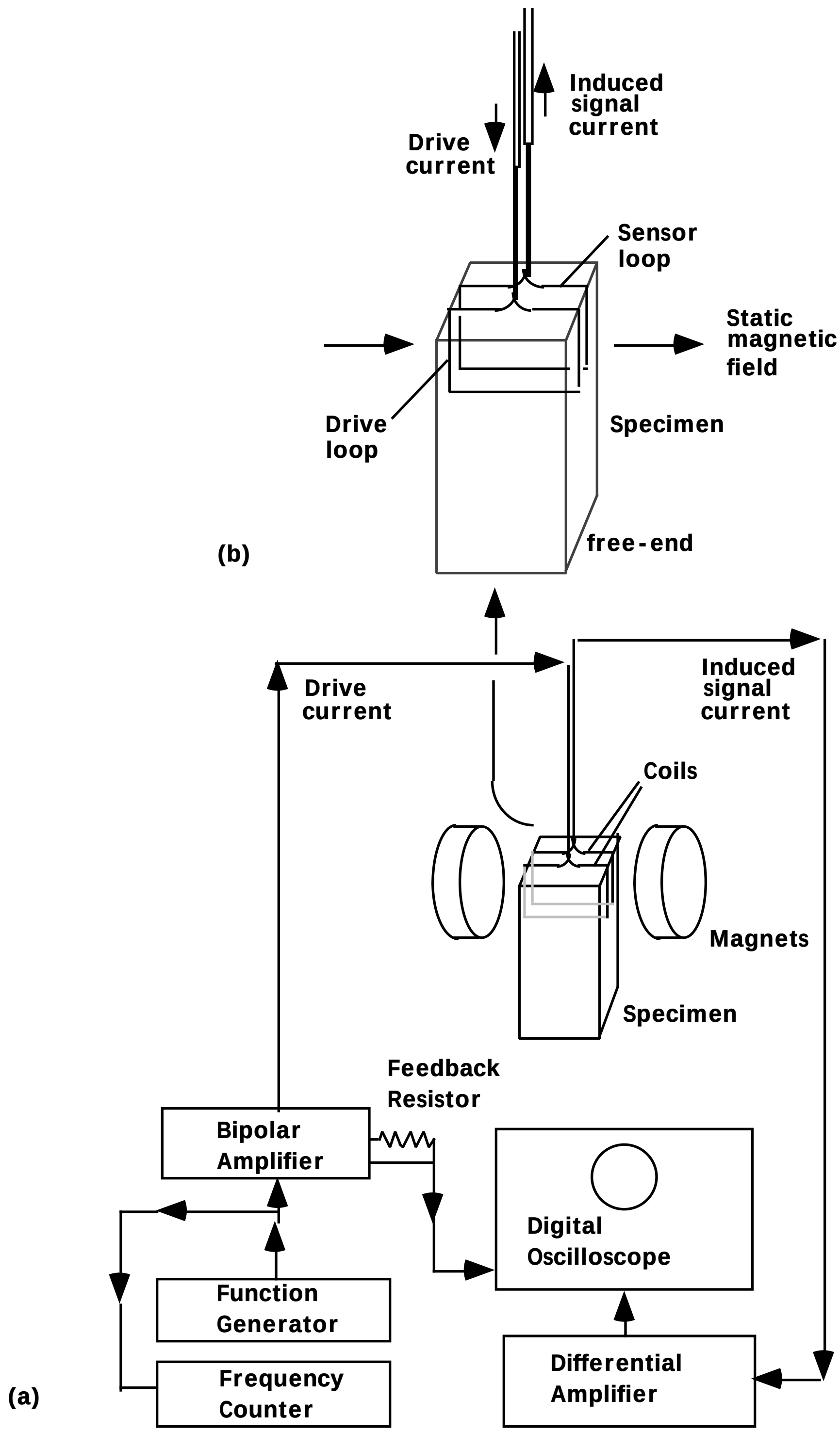
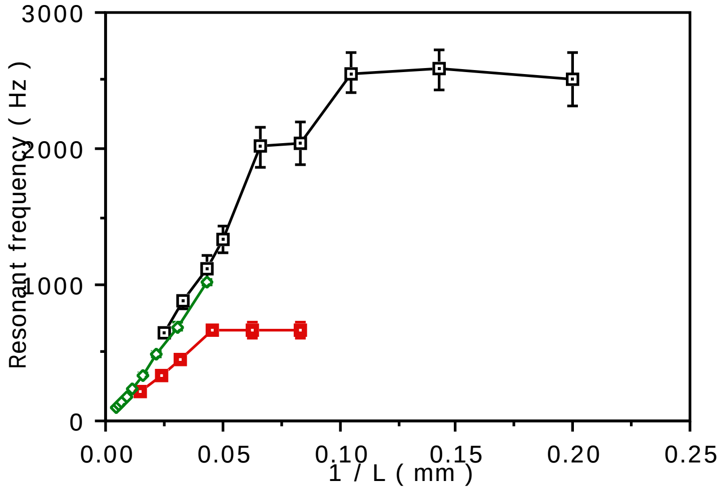


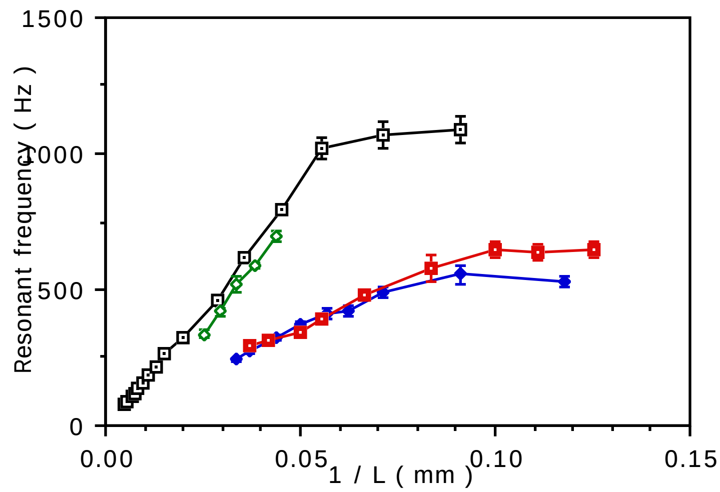
1. Schematic diagram of free-free torsion resonance experiment apparatus.

(a) Apparatus schematic.

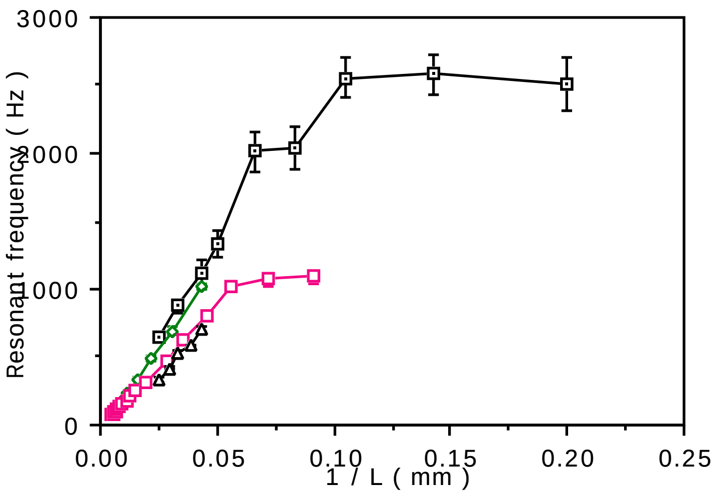
(b) Detail of specimen.



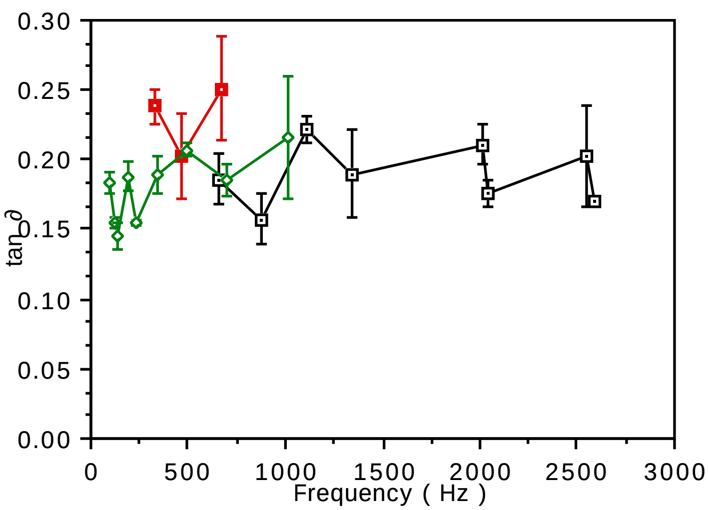
2. Fundamental resonant frequency ν against inverse of the specimen length, polyester foam. ⁄: conventional foam, diameter=12.7mm. ◊: conventional foam, width=25.4mm. ⁄: re-entrant foam, width=25.4mm.



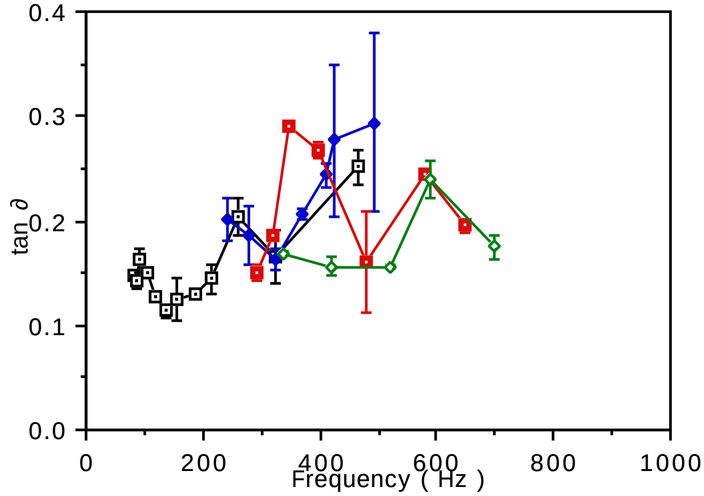
3. Fundamental resonant frequency ν against inverse of the specimen length, Scott industrial foam. ⁄: conventional foam, width=25.4mm. ◊: conventional foam, diameter=9mm. ⁄: re-entrant foam, diameter=13.5mm. ◊: re-entrant foam, width=15mm.



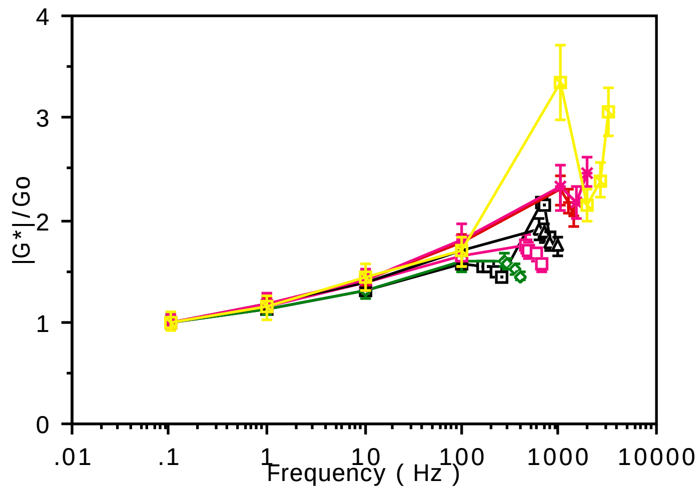
4. Fundamental resonant frequency ν against inverse of the specimen length. ⁄: conventional polyester foam, diameter=12.7mm. ◊: conventional polyester foam, width=25.4mm. ⁄: conventional Scott industrial foam, width=25.4mm. Δ: conventional Scott industrial foam, diameter=9mm.



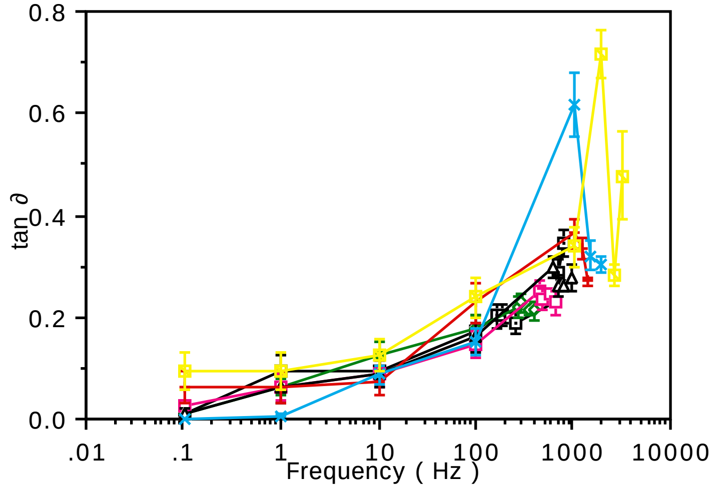
5. Loss tangent tan δ of polyester foam. ⁄: conventional foam, diameter=12.7mm. ◊: conventional foam, width=25.4mm. ⁄: re-entrant foam, width=25.4mm.



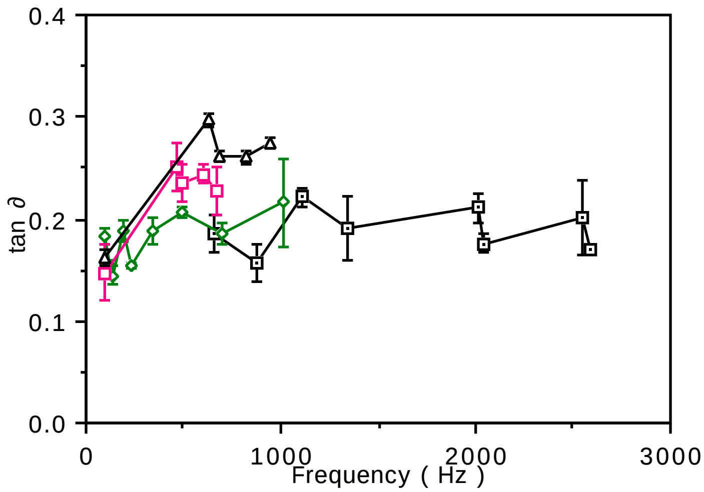
6. Loss tangent tan δ of Scott industrial foam. ⁄: conventional foam, width=25.4mm. ◊: conventional foam, diameter=9mm.⁄: re-entrant foam, diameter=13.5mm. ◊ : re-entrant foam, width=15mm.



7. Normalized stiffness |G*|/Go of conventional polyester foam, diameter=12.7mm. Length L, hence resonant frequency varies.⁄: L=47.5mm, ◊: L=27mm,⁄: L=15mm, Δ: L=10.5mm,+ : L=6mm, x: L=4mm, and **fl**: L=3mm.



8. Loss tangent tan δ of conventional polyester foam, diameter=12.7mm. Length L, hence resonant frequency varies. ⁄: L=47.5mm, ◊: L=27mm, ⁄: L=15mm, Δ: L=10.5mm, +: L=6mm, x: L=4mm, and **fl**: L=3mm.



9. Loss tangent tanδ of conventional polyester foam. Comparisons of tanδ obtained from different experimental methods. ⁄: free-free torsion resonance experiment, diameter=12.7mm. ◊: free-free torsion resonance experiment, width=25.4mm. ⁄: micromechanics apparatus, diameter=12.7mm, length=15mm. Δ: micromechanics apparatus, diameter=12.7mm, length=10.5mm.

TABLE 1
Specimen dimensions and cut-off frequencies

Type of foam	Compression ratio	Specimen size (diameter or width)	Poisson's ratio	Cut-off frequency	| G* | at a representative frequency	Wave dispersion constant
Conventional polyester foam 0.5mm cells	N/A	d = 12.7mm	0.3	2500 Hz (650 Hz)	81 kPa	0.089
re-entrant polyester foam ≈0.35mm cells	2.89	w = 25.4mm	-0.6	700 Hz (300 Hz)	81 kPa	0.03
Conventional Scott industrial foam 2.5mm cells	N/A	w = 25.4mm	0.3	1000 Hz (300 Hz)	33 kPa	0.28
re-entrant Scott industrial foam ≈1.7mm cells	3.25	d = 13.5mm	-0.5	650 Hz (300 Hz)	21 kPa	0.27
re-entrant Scott industrial foam ≈1.6mm cells	3.56	w = 15mm	-0.3 (300 Hz)	530 Hz	29 kPa	0.19

Table 2.

Effective shear modulus |G*| at 0.1 Hz of conventional polyester foam (diameter=12.7mm).

Length(mm)	|G*|(kPa)
47.5	54
27.	54
15.	60
10.5	65
6.	82
4.	79
3.	120