Lakes, R. S., "Design considerations for negative Poisson's ratio materials"*ASME Journal of Mechanical Design*, 115, 696-700, (1993).

Abstract

This article presents a study of the implications of negative Poisson's ratios in the design of components subjected to stress. When the Poisson's ratio becomes negative, stress concentration factors are reduced in some situations and unchanged or increased in others. Stress decay according to Saint Venant's principle can occur more or less rapidly as the Poisson's ratio decreases. Several design examples are presented, including a core for a curved sandwich panel and a flexible impact buffer.

1 Introduction

Recently, the invention of negative Poisson's ratio foams was reported[1-3]. Foam materials based on metals and several polymers were transformed so that their cellular architecture became re-entrant, i.e. with inwardly protruding cell ribs. Foams with re-entrant structures exhibited negative Poisson's ratios as well as greater resilience than conventional foams. The polymer foams exhibited negative Poisson's ratios as small as -0.7, and values to -0.8 have been observed in metal foams.

In the theory of elasticity, it is well known that some solutions for stress or displacement fields are dependent on Poisson's ratio. However, since most engineering materials have Poisson's ratios very close to 0.3, such dependence has received comparatively little attention. With the advent of new materials which exhibit controllable negative values of substantial magnitude, the effect of Poisson's ratio on stress and displacement fields deserves re-examination within the context of design. Applications of materials with negative Poisson's ratios may be envisaged (i) based on the Poisson's ratio, (ii) based on the superior toughness and tear resistance which has been observed in these materials, and (iii) based on the acoustic properties associated with the vibration of ribs in the material. In this article we consider the implications of the negative Poisson's ratio in design and in new applications.

2 Consequences of negative Poisson's ratio

Many phenomena in the deformation of elastic materials depend on the Poisson's ratio. The simplest is that a material with a negative Poisson's ratio will get fatter in cross section when stretched and thinner when compressed. The relations between the shear modulus G, the Young's modulus E, the bulk modulus B (the inverse of the compressibility) and Poisson's ratio ν are: $B = 2G(1+\nu)/3(1-2\nu)$, and $E = 2G(1 + \nu)$. When the Poisson's ratio approaches 1/2, as in rubbery solids, the bulk modulus greatly exceeds the shear modulus and the material is referred to as incompressible. When the Poisson's ratio approaches -1, the material becomes highly compressible but difficult to shear; its bulk modulus and Young's modulus are much less than its shear modulus.

Poisson's ratio has no effect on stress distributions in many two-dimensional situations however it may have a dramatic effect in three dimensions. As for two dimensions, Michell's theorem [4] states that the two-dimensional stress field is independent of Poisson's ratio if either the body is singly connected or tractions over each interior boundary give no net force. If either of these conditions is not met, there can be a large effect of Poisson's ratio even in two dimensions [5]. Some three dimensional stress distributions, including many involving torsion, are independent of Poisson's ratio.

Stress concentrations

Stress concentrations may depend upon Poisson's ratio ν. In most two dimensional situations, the stress concentrations have no dependence at all on Poisson's ratio. In three dimensions, however, there may be a significant dependence of the stress concentration factor upon Poisson's ratio. Specific examples for spherical and cylindrical cavities and rigid inclusions are as follows [6]. Stress concentration factors are given for points at the pole or equator of the inhomogeneity. The pole is in the direction of the applied load; the equator of the inclusion is the set of points on it intercepted by the plane perpendicular to the applied load. The cylinder long axis is perpendicular to the applied load. The $\theta\theta$ direction considered at the equator for the spherical cavity corresponds to the direction of the applied load; the $\psi\psi$ direction is orthogonal to it. For the spherical cavity the stress concentration factors for the equator in the $\theta\theta$ and $\psi\psi$ directions are positive for ordinary Poisson's ratio, and the usually quoted value for the $\theta\theta$ direction is larger in magnitude. However the stress concentration factor for the $\psi\psi$ direction changes sign for small Poisson's ratio. The stress concentration factor for shear is based on a shear stress at the pole, in a direction orthogonal to the plane of the shear.

Spherical cavity in uniaxial tension

$$SCF|_{polar} = -\frac{3+15\nu}{14-10\nu} \quad SCF|_{eq\theta\theta} = \frac{27-15\nu}{14-10\nu} \quad SCF|_{eq\psi\psi} = -\frac{3-15\nu}{14-10\nu}$$

Spherical cavity in biaxial tension

$$SCF = \frac{12}{7-5\nu}$$

Spherical cavity in pure shear

$$SCF = \frac{15(1-\nu)}{7-5\nu}$$

Rigid cylindrical inclusion in uniaxial tension

$$SCF|_{polar} = \frac{1}{2}\left(3-2\nu+\frac{1}{3-4\nu}\right)$$

$$SCF|_{eq} = \frac{1}{2}\left(1+2\nu-\frac{3}{3-4\nu}\right)$$

Rigid spherical inclusion in uniaxial tension

$$SCF|_{polar} = \frac{2}{1+\nu} + \frac{1}{4-5\nu}$$

$$SCF|_{eq} = \frac{\nu}{1+\nu} - \frac{5\nu}{8-10\nu}$$

Rigid spherical inclusion in hydrostatic tension

$$SCF|_{radial} = 3\,\frac{1-\nu}{1+\nu}$$

For the spherical inclusion the stress concentration factor is the same for equatorial hoop and axial stresses. Stress concentration factors as they depend on Poisson's ratio are plotted in Fig. 1 for cavities and in Fig. 2 for inclusions. Observe that a negative Poisson's ratio reduces the stress concentration factor for some of the situations involving cavities, and increases it for others. Stress around a cavity in a field of hydrostatic stress is not affected by Poisson's ratio. For rigid inclusions, the stress concentration factor becomes large as ν approaches -1.

Ellipsoidal cavities are also of interest in that they can generate much larger stress concentration factors than spherical ones. The relationships for prolate [7] and oblate [8] ellipsoidal cavities are rather complicated. We calculate the stress concentrations for the locations shown for the full range of Poisson's ratio and display them for prolate ellipsoids in Fig. 3, and for oblate ellipsoids in Fig. 4. For uniaxial tension, the usual stress concentration for σ_1 does not vary very dramatically with Poisson's ratio, even for very flat ellipsoids (a/b >> 1) for which the stress concentration factors become large; σ_2 shows more variation with ν. Given stress concentration factors are for the equatorial region, in the direction of the applied load. This location and orientation gives maximum stress under ordinary conditions but it is not guaranteed to be maximal for all Poisson's ratios.

The toughness of a material with sharp cracks can also depend on its Poisson's ratio. In the context of elasticity theory, the critical tensile stress [9] σ for fracture of a solid of surface tension T, Young's modulus E, with a plane circular crack of radius r is $\sigma = \sqrt{[\pi ET/2r(1-\nu^2)]}$. When the Poisson's ratio approaches -1, a material of given Young's modulus is predicted to become very tough. However,

$$E/(1-\nu^2) = 2G(1+\nu)/(1+\nu)(1-\nu) = 2G/(1-\nu).$$

So the toughness for given shear modulus does not diverge, but for $\nu \to -1$, G becomes large in comparison with E.

The toughness of a material also depends upon its nonlinear properties, as well as upon structural aspects. In the materials with negative Poisson's ratios described by the author [1-3], the presence in these materials of many curved ribs with low stiffness but high extensibility may be expected to increase the toughness; measurements of toughness will be reported elsewhere.

Saint Venant's principle

An important problem in solid and structural mechanics is the question of the distance which local stress distributions penetrate into structural members. In many practical problems in mechanics, one appeals to Saint Venant's principle in neglecting end effects or edge effects at some distance from the boundaries of the structural element. Saint Venant's principle is usually demonstrated for particular geometries by the development of energy inequalities. However, some exact three dimensional solutions are available for the decay of stress due to a self equilibrated stress distribution on a boundary; many such stress distributions depend on Poisson's ratio.

Some examples of the dependence of Saint Venant stress decay on Poisson's ratio include the following (Fig. 5). In a pretwisted shell [10] of half-width w, thickness h, and pretwist per length β, the stress decays with distance from the end as $e^{-z/L}$ and the decay length L for end stresses is $L = \Lambda b = 12(1-\nu^2)^{1/2} \beta b^2/h$, so that as Poisson's ratio approaches -1, the decay length tends to zero. The decay length L of end effects (determined by energy methods) in the torsion of a narrow rectangular cross section bar with the warp restrained at one end is $L = \mathbf{a}[(1+\nu)/5]^{1/2}$, in which **a** is the bar width [11], so that the decay length decreases as Poisson's ratio becomes smaller. The decay length is smaller in the case of a square cross section bar, but depends on Poisson's ratio in a similar way as shown in Fig. 5. As for self-equilibrated axisymmetric loads on the end of a circular cylinder, the effect of Poisson's ratio is predicted [12] to be opposite: the decay length increases as Poisson's ratio becomes smaller. Some caution is needed in viewing these results [12] since they were arrived at by energy approximations rather than exact solutions, so it is not guaranteed that the approximations are uniform with Poisson's ratio. Indeed, recent study of the exact eigenvalue problem indicates that slow stress decay occurs in circular cylinders with self-equilibrated end loads for Poisson's ratios approaching -1 [13]. As shown in Fig. 5, the two predictions agree over the normal range of Poisson's ratio but disagree for values approaching -1.

As for normal loads upon an infinite elastic half space [14], the decay of stress goes as a power law rather than an exponential. The power in the power law does not depend on ν, but the magnitude of the mean normal stress for all distances from the load goes as $1+\nu$, so that this stress vanishes as ν tends to -1.

Other stress distributions

Consider cantilever bending of a beam of circular cross section of radius a, with the centerline in the z direction. Bending is by a concentrated load P in the x direction. Axial and shear stress components are [11]

$$\sigma_{zz} = - \frac{4P}{\pi a^4} (L - z)x$$

$$\sigma_{xz,\ surf} = - \frac{P}{\pi a^2} \frac{1+2\nu}{1 + \nu}$$

$$\sigma_{xz,center} = - \frac{P}{\pi a^2} \frac{(3+2\nu)}{2(1 + \nu)}$$

$$\sigma_{yz,center} = 0$$

As Poisson's ratio approaches -1, the σ_{xz} shear stresses become large, the shear modulus becomes large, but the deflection curve is unchanged. For a Poisson's ratio of -0.5 the surface shear stress σ_{xz} vanishes, while the center shear stress does not vanish for any realizable value of ν. The shear at the center is equal or larger in magnitude than the shear at the surface for positive Poisson's ratios, but for sufficiently small negative Poisson's ratios the shear at the surface becomes larger. For $\nu = -1$, the surface shear is larger by a factor of two. So, we cannot always assume that the location of the maximum stress is always the same when Poisson's ratio can vary over the full range.

In the case of a uniformly distributed load [11], the curvature K depends on Poisson's ratio:

$$K = \frac{M}{EI} \left[1 - \frac{\{7 + 12\nu + 4\nu^2\}}{6(1+\nu)} \frac{a^2}{L^2}\right] ,$$

such that for $\nu \rightarrow -1$, the bend rigidity M/K becomes small.

In the indentation of a block of material by a localized circular pressure distribution of radius a, consider the indentation rigidity is $P/w = E/2a(1- \nu^2)$, in which E is Young's modulus, P is the pressure, and w is the indentation [11]. A material with a negative Poisson's ratio approaching the thermodynamic limit -1 will be difficult to indent even if the material is compliant in tension/compression. Since $E/(1- \nu^2)= 2G/(1- \nu)$ it is evident that stiffening effects of this type reflect the relationship between Poisson's ratio and the shear modulus.

Transverse curvature in bending

Consider the transverse curvature which occurs in the pure bending of a beam or a plate. When Poisson's ratio is positive, the transverse curvature is opposite the principal curvature of bending and is known as the anticlastic curvature. For a material with a negative Poisson's ratio, transverse curvature is in the same sense as the principal curvature: synclastic curvature, as is anticipated in the exact, three-dimensional solution for the problem of pure bending [11]. Synclastic (or conclastic) curvature is demonstrated in Fig. 6 for a re-entrant honeycomb, which is bent by moments along two of its four edges.

3 Design examples

Core for sandwich panel

Honeycombs with cells shaped as regular hexagons are commonly used as lightweight cores for flat sandwich panels. Such honeycombs are anisotropic, with high stiffness and a Poisson's ratio near zero for out-of plane deformation; and Poisson's ratios ν near +1 for the in-plane directions. Bending of such a honeycomb gives rise to a saddle shape as a result of the anticlastic curvature. Such conventional honeycombs strongly resist being bent into any shape other than the hyperboloid which results from anticlastic curvature. In particular the rigidity (moment divided by curvature) for the bending of an isotropic plate free to undergo anticlastic curvature is $Eh^3/12$; for a plate bent to a cylindrical shape it is $Eh^3/12(1-\nu^2)$; for a plate bent to a spherical shape it is $Eh^3/12(1-\nu)$. For conventional honeycomb (which is highly anisotropic), $\nu \approx +1$ for the in-plane deformation associated with bending, so that for bending into a spherical or cylindrical shape the rigidity diverges. Attempts to forcibly bend them into a convex shape cause the cell walls to buckle.

Bendable honeycombs which can be formed into convex shapes are currently available under the trade name Flex-core®, of Hexcel Co, as shown in Fig. 7. This material is anisotropic in-plane: properties measured in our laboratory for a representative sample of density 0.08 g/cm^3 were: in-plane Poisson's ratios $\nu_{21} = +0.5$ and $\nu_{12} = +0.9$, and Young's moduli $E_1 = 0.4$ MPa, and $E_2 = 0.2$ MPa. This contrasts with honeycomb of regular hexagons for which both in plane Poisson's ratios are +1, and the out of plane Poisson's ratio is ≈0. It is possible to bend Flex-core® honeycomb into a convex shape, but considerable force is required to achieve a small radius of curvature.

By contrast, a negative Poisson's ratio honeycomb [15-17] can be prepared with inverted hexagonal cells. It is easily bent into a convex shape such as that shown in Fig. 6 with a minimum of bending moment. Such a core, when combined with stiff face material, could be used to make convex curved sandwich panels. The convex shape would be spherical for $\nu = -1$ (which is possible in a re-entrant honeycomb), cylindrical for $\nu = 0$, and ellipsoidal for $-1 < \nu < 0$. Cell shapes for negative Poisson's ratio honeycomb are shown in Fig. 6, and for comparison, an idealized cell shape for negative Poisson's ratio foam is shown in Fig. 8. Sandwich panels of convex shapes may find use in the aircraft industry, in the construction of domes, and elsewhere.

Sandwich panel cores are also made from stiff three dimensional cellular solids (foams) which exhibit $\nu \approx 1/3$. The foams are isotropic or nearly so, unlike honeycombs. Since the in-plane stiffness is comparable to the out of plane stiffness (which should be high) it is not practical to bend a flat foam sheet into a curved shape. However in the case of polymer foam cores, such bending could be achieved while the polymer is in the process of polymerization, or in thermoplastic polymers polymers temporarily softened by heat. Forming of the core into a convex shape by this procedure would be facilitated by the use of negative Poisson's ratio foam. Other properties of such foam, such as toughness, are currently under study.

Wrestling mat

Consider the choice of materials for an elastic impact force buffer such as a wrestling mat or a knee pad. Let us examine the penetration rigidity F/u in which F in the indentation force and u is the maximum displacement, for elastic materials. It is desired that the buffer should be effective for both large and small impactors. In the context of a wrestling mat, either an elbow or a person's entire back may strike the mat. For a sufficiently small impactor, approximate the mat as an elastic half space under a circular pressure distribution [11] of radius a,

$$[F/u]_{narrow} = Ga_n/(1-\nu).$$

For an impactor much larger than the mat thickness, consider the uniform compression of a layer of thickness H and radius a_w, in which the lateral Poisson effect is restrained. Let the force F be distributed uniformly over the layer. Then,

$$[F/u]_{wide} = \frac{Ga_w^2}{\frac{2H(1+\nu)}{(1-2\nu)}} \quad .$$

In this application, it is desired that the layer reduce impact forces which may be distributed over a wide area or a narrow area. The layer must be sufficiently compliant for distributed forces, yet must be sufficiently rigid that it does not bottom out under a concentrated force. The following ratio should be of the order of unity.

$$[F/u]_{wide}/\,[F/u]_{narrow} = \frac{2a_w^2}{a_nH}\;\frac{1-\nu^2}{1-2\nu} \quad .$$

Materials with negative Poisson's ratios offer the best performance, and rubbery materials are the worst in this application. Current mats tend to be made of elastomeric foams (with $\nu \approx 1/3$) rather than rubber(with $\nu \approx 1/2$), a logical choice in view of the above. As an example, let $a_w = 10H$, and $a_n = 0.1H$ (narrower than an elbow but consistent with the approximation made), so $a_w/a_n = 100$. Then the optimal Poisson's ratio is $\nu = -0.9993$. We remark that an exact solution is available for the stresses in an elastic layer acted upon by a distribution of force [18], however such complexity is not necessary in this example.

4 Discussion and conclusions

As for fabrication, negative Poisson's ratio honeycombs can be made from conventional ones by mechanically inverting each cell, provided that the cell walls are sufficiently flexible. They can also be made ab initio by preparing corrugated strips and cementing them. As for the relationship between cell shape and Poisson's ratio, theoretical treatments are available of deformation of hexagonal cell honeycombs, including those with re-entrant cells giving rise to negative Poisson's ratios [15-17]. The high out-of plane stiffness associated with conventional honeycomb is retained in honeycomb with a negative Poisson's ratio.

Many stress distributions relevant to design are significantly affected when the Poisson's ratio of the material assumes negative values, particularly values approaching the lower limit -1. Stress concentrations can be very sensitive to Poisson's ratio; negative values of Poisson's ratio can be helpful or harmful, depending on the context. Published stress concentrations are for stress components which are maximal for $\nu \approx 0.3$. For materials with negative Poisson's ratios, other stress components may be more important. For example in the case of the rigid inclusion, stress at the pole can have large values of opposite sign to the stress at the equator usually considered. Such a stress could be very important in a brittle material with a negative Poisson's ratio. Moreover, in cantilever bending, the location of the maximum shear stress changes as Poisson's ratio is made sufficiently negative. Consequently, it is recommended that all stress components be considered in design-oriented calculations for negative Poisson's ratio materials. Elastomeric or ductile materials with negative Poisson's ratios may not be as sensitive to stress concentrations as are brittle ones. However there are many structural possibilities for negative Poisson's ratio materials [19]. Recently, laminate structures have been introduced which give rise to negative Poisson's ratios [20]. Other aspects of Poisson's ratio such as wave propagation and reflection problems have been recently treated [21]; these may be relevant to ultrasonic or geological applications.

Negative Poisson's ratio foams and composites are not homogeneous. The application of the continuum theory of elasticity may therefore be questioned. Phenomenologically it is observed that in fibous composites and in compact bone, stress concentration factors, determined in the laboratory for holes which are not very much larger than the structure size, are smaller than values calculated via elasticity theory. Some writers attempt to circumvent this disagreement by calculating an average stress in a region rather than using the maximum stress to predict failure. The influence of the microstructure size can be incorporated more rationally via the Cosserat theory of elasticity [22-25], which, unlike the classical theory, has a natural length scale. In Cosserat solids, the stress concentration factor around an inhomogeneity is predicted to differ [24] from classical values if the inhomogeneity size is comparable to the the characteristic length ” of the material. Phenomena associated with Cosserat elasticity are likely to be of larger magnitude, and therefore of greater interest in materials such as composites and cellular solids with comparatively large structural features. In cellular solids, the characteristic length ” may be the on the order comparable to the average cell size. The physical origin of such effects is in the bending and twisting moments transmitted through the fibers in a composite or in the cell ribs in a foam.

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6 References

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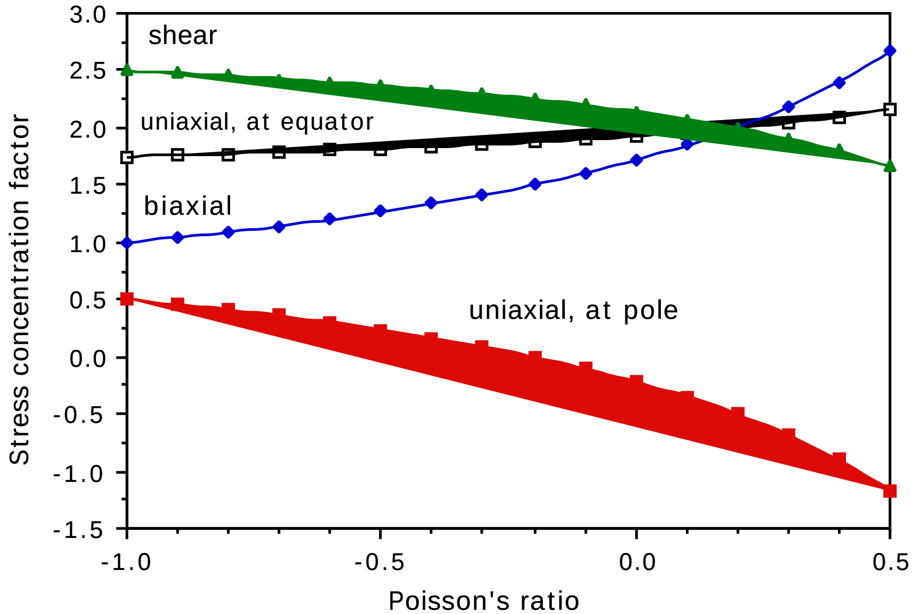
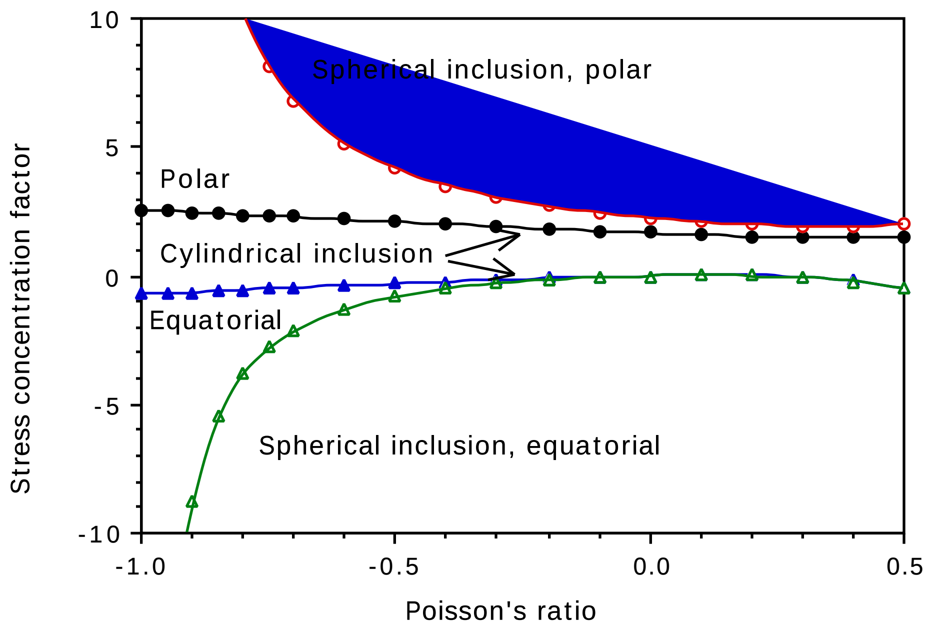
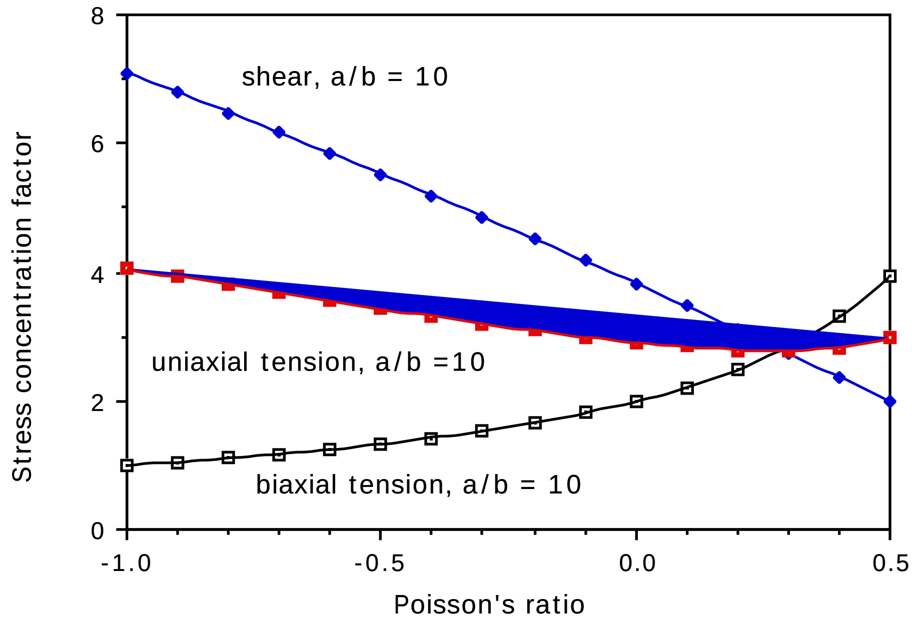
Figures



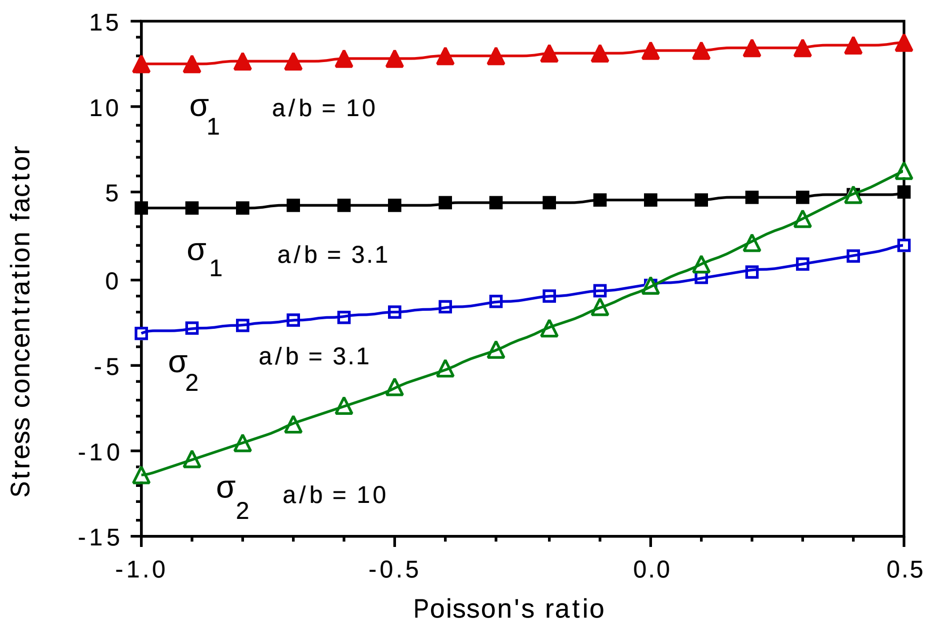
1. Stress concentration factors vs Poisson's ratio for a spherical cavity. Uniaxial tension, biaxial tension, and shear at the equator; uniaxial tension at the pole.



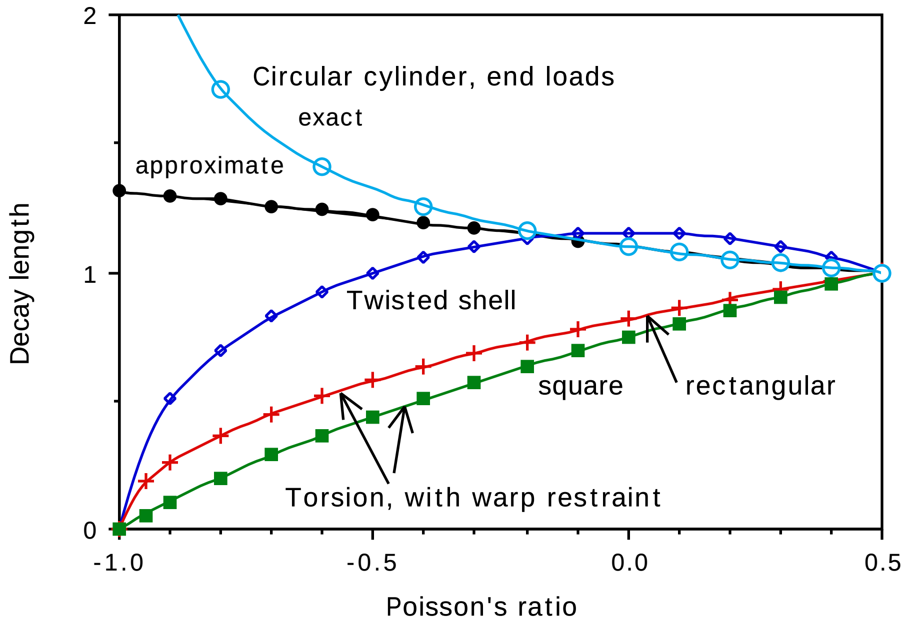
2. Stress concentration factor vs Poisson's ratio for rigid spherical and cylindrical inclusions under uniaxial tension.



3. Stress concentration factor vs Poisson's ratio for a prolate ellipsoidal cavity under uniaxial tension, shear, and biaxial tension, for a/b = 10.



4. Stress concentration factor vs Poisson's ratio for an oblate ellipsoidal cavity under uniaxial tension, for a/b = 3.1 and for a/b =10.



5. Illustration of stress decay according to Saint-Venant's principle. Normalized decay length for stress vs Poisson's ratio for different situations. For the end effects in circular cylinders, the approximate solution, based on an energy approach, is from [11]; the exact solution is from [12].

6. Bending of a negative Poisson's ratio honeycomb.
See video in http://silver.neep.wisc.edu/~lakes/Poisson.html

7. Commercially available honeycomb, Flex-core®, which can be deformed with some effort into a convex shape.

8. Idealized unit cell of negative Poisson's ratio foam.
See Lakes, R. S. "Foam structures with a negative Poisson's ratio", *Science*, 235 1038-1040 (1987).