Micromechanical analysis of dynamic behavior of conventional and negative Poisson's ratio foams

by

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Abstract

Both conventional and negative Poisson's ratio foams exhibit dispersion of acoustic waves as well as cut-off frequencies at which the group velocity tends to zero. This macroscopic behavior is attributed to micro-vibration of the foam cell ribs. The purpose of this article is to develop a micromechanical model of the cut-off frequency. This model is based on the resonance of ribs which may be straight, curved, or convoluted. As foam cell ribs become more curved, the cut-off frequency decreases. Therefore foams with curved or convoluted ribs are expected to provide superior performance in applications involving the absorption of sound.

1. Introduction

Acoustic waves are dispersed as they propagated through a fiber-reinforced viscoelastic material [1]. For three-dimensional random particulate composite containing spherical inclusions, the wave propagation behavior is significantly influenced by the excitation of the particle resonances [2]. Cellular solids also exhibit dispersion of acoustic waves, in which the wave speed varies with frequency, and cut-off frequencies, in which the phase velocity of acoustic waves tends to zero as frequency increases [3]. These effects become more pronounced and occur at lower frequency [4] in re-entrant foams with negative Poisson's ratio [5]. Moreover, negative Poisson's ratio polymer foams exhibit superior sound absorption in comparison with polymer foams of conventional structure [6]. An increase in wave speed with frequency is associated with viscoelastic behavior [7] or with effects due to rotational motion of the microstructure [8]. A decrease in wave speed with frequency (as well as cut-off frequencies) is associated with micro-vibration of the structural elements [9]. In this article we present an analytical model of the vibration of structural elements in cellular solids with the aim of predicting the cut-off frequency of acoustic waves in these materials. It is intended that the understanding gained will be of use in the design and use of materials which offer superior performance in the absorption of sound or vibration.

2. Method

The foam micro-resonance behavior is analyzed by considering the foam rib as a vibrating member, which is modeled to be a curved beam, a partial ring, or a helical spring as the convolutedness of the member increases. The cut-off frequency of the foam is derived from the relation for the lowest vibrational mode of the rib and from a relation extracted from the theory of elasticity of open-cell foams.

In the following, the natural frequency of a foam rib modelled according to various geometrical assumptions is presented. The natural frequency of such ribs depends on the boundary conditions at the ends. The ends may be fixed or free to move in idealized analyses. In an actual foam, each rib is connected to other ribs, so it is neither fixed nor free but somewhere in between. The difference between fixed and free conditions is typically a factor of two; this is much less than the factor of hundreds associated with the difference in curvature or convolutedness of the ribs.

2.1. Foam rib modeled as a curved beam (sector angle 0° ~ 90°)

The natural angular frequencies ω_n of a complete circular ring, have been shown to be [10]

$$\omega_n = \sqrt{\frac{r^2 E_s}{4R^4 \rho_s} \; \frac{n^2(n^2-1)^2}{n^2+1}} \tag{1}$$

(circular cross section),

or

$$\omega_n = \sqrt{\frac{I\ E_s}{AR^4 \rho_s} \; \frac{n^2(n^2-1)^2}{n^2+1}} \tag{2}$$

(rectangular cross section)

for flexural vibration in the plane of the ring,

and

$$\omega_n = \sqrt{\frac{r^2 E_s}{4R^4 \rho_s} \; \frac{n^2(n^2-1)^2}{n^2+1+\nu}} \tag{3}$$

(circular cross section)

for flexural vibration normal to the plane of the ring, in which ω_n is the natural angular frequency in radians per second, r is the radius of the ring's circular cross section, R is the radius of the ring, E_s and ρ_s are the Young's modulus and mass density of the solid ring, A is the area and I is the moment of inertia of a rectangular cross section, n is an integer used to define the vibration shape of the ring (2n = number of nodal points; $n \geq 2$,), and ν is the Poisson's ratio of the ring material.

It can be easily determined from Eqs. (1) and (3) that the natural frequencies of a ring vibrating flexurally in plane and normal to plane differ by less than 3% with $\nu = 0.3$. Therefore, Eqs. (1) and (2) are used to predict the flexural vibration of a ring with the circular cross section, and the rectangular cross section respectively.

The fundamental vibration of a curved beam simply supported at both ends is mechanically equivalent to the flexural vibration of a segment between two nodal points of a complete ring. The natural frequency of a curved beam of sector angle θ ($0° \leq \theta \leq 90°$) are thus obtainable from Eqs. (1) and (2) with

$$n = \frac{180°}{\theta} \tag{4}$$

or $$n = \frac{\pi R}{L} \tag{5}$$

in which L is the length of this curved beam representing the foam cell rib. The natural frequency ω_{beam} (in radians per second) of a curved beam simply supported at both ends is also obtained as

$$\omega_{beam} = \frac{\pi^2}{2} \frac{r}{L^2} \sqrt{\frac{E_s}{\rho_s}} \frac{n^2-1}{n} \sqrt{\frac{1}{n^2+1}} \tag{6}$$

(circular cross section)

by substituting Eq. (5) into Eq. (1),

or $$\omega_{beam} = \frac{\pi^2}{\sqrt{12}} \frac{t}{L^2} \sqrt{\frac{E_s}{\rho_s}} \frac{n^2-1}{n} \sqrt{\frac{1}{n^2+1}} \tag{7}$$

(rectangular cross section)

by substituting Eq. (5) into Eq. (2) with $A = mt^2$ and $I = mt^4/12$, in which the dimensions of the rectangular cross section are t by mt and $m \geq 1$.

The natural frequency ω_{beam} is determined by the boundary conditions as well. The natural frequency of a straight beam with clamped-clamped ends or free-free ends is 2.25 times as large as that simply supported at both ends [9]. Therefore, the natural frequency of a curved beam with clamped-clamped ends or free-free ends is obtained as

$$\omega_{beam} = \frac{2.25\pi^2}{2} \frac{r}{L^2} \sqrt{\frac{E_s}{\rho_s}} \frac{n^2-1}{n} \sqrt{\frac{1}{n^2+1}} \tag{8}$$

(circular cross section),

or $$\omega_{beam} = \frac{2.25\pi^2}{\sqrt{12}} \frac{t}{L^2} \sqrt{\frac{E_s}{\rho_s}} \frac{n^2-1}{n} \sqrt{\frac{1}{n^2+1}} \tag{9}$$

(rectangular cross section)

by multiplying Eqs. (6) and (7) by a factor of 2.25. Application of eqs. (8) and (9) to foam rib mechanics is approximate since the factor of 2.25 between different boundary conditions of a straight beam is applied to the case of a curved beam.

For an open-cell foam, analysis of the overall foam material stiffness based on rib bending in the theory of elasticity predicts [11,12]

$$\frac{E_s}{\rho_s} = \frac{1}{4} \frac{E}{\rho} \frac{L^2}{r^2} \tag{10}$$

for circular cross section ribs,

and $$\frac{E_s}{\rho_s} = \frac{2}{m^2+1} \frac{E}{\rho} \frac{L^2}{t^2} \tag{11}$$

for rectangular cross section ribs,

in which E_s and ρ_s are the Young's modulus and mass density of the solid of which the foam is made, E and ρ are the Young's modulus and mass density of the foam, and foam rib properties r,t,m, and L are as previously defined.

Substituting Eqs. (10) and (11) into Eqs. (6)-(9), and recognizing that $\omega_{beam} = 2\pi\nu_{cut}$, the rib resonance frequency ν_{rib} (Hz), hence the cut-off frequency of open-cell foams ν_{cut} (in Hz) on the basis of simply supported ribs can be obtained as

$$\nu_{cut} = 0.393 \frac{1}{L} \sqrt{\frac{E}{\rho}} \frac{n^2-1}{n} \sqrt{\frac{1}{n^2+1}} \tag{12}$$

(circular cross section),

or $$\nu_{cut} = 0.641 \frac{1}{\sqrt{m^2+1}} \frac{1}{L} \sqrt{\frac{E}{\rho}} \frac{n^2-1}{n} \sqrt{\frac{1}{n^2+1}} \tag{13}$$

(rectangular cross section),

and the cut-off frequency of open-cell foams ν_{cut} (in Hz) on the basis of clamped-clamped end or free-free end ribs can also be obtained as

$$\nu_{cut} = 0.884 \frac{1}{L} \sqrt{\frac{E}{\rho}} \frac{n^2-1}{n} \sqrt{\frac{1}{n^2+1}} \tag{14}$$

(circular cross section),

or $$\nu_{cut} = 1.44 \frac{1}{\sqrt{m^2+1}} \frac{1}{L} \sqrt{\frac{E}{\rho}} \frac{n^2-1}{n} \sqrt{\frac{1}{n^2+1}} \tag{15}$$

(rectangular cross section).

We define a parameter Ξ associated with the degree of convolutedness of the ribs. Several rib geometries are considered here, and the purpose of Ξ is to provide a common representation for the different analytical solutions used.

The convolutedness parameter Ξ of a curved beam of sector angle θ is defined as follows:

$$\Xi = \frac{L}{\ell} = \frac{\theta \frac{\pi}{180}}{2 \sin(\frac{\theta}{2})} \tag{16}$$

in which ℓ is the straight distance between two end points of the curved beam. The value of Ξ increases from 1 as a straight beam to 1.11 as θ = 90°.

2.2. Foam rib modeled as a partial ring (sector angle 180° ~ 360°)

Using the Rayleigh method, the natural frequency of a partial ring of sector angle θ between 180° and 360° with both ends clamped can be obtained [13]. The fundamental angular frequency ω_{ring} (in radians per second) of vibration of such a ring in its own plane is given as

$$\omega_{ring} = f(\theta) \sqrt{\frac{E_s I}{\mu R^4}} \tag{17}$$

in which $f(\theta)$ is a numerical factor which decreases as θ increases ($f(\theta)=4.5$ at $\theta=180°$ and $f(\theta)=0.6$ at $\theta=360°$), E_s is Young's modulus of the solid from which the ring element considered as a foam rib is made, and μ is the mass density per unit length of the ring.

Substituting Eqs. (10) and (11) into Eq. (17), with $I=r^4\pi/4$ and $\mu=\rho_s r^2\pi$ for circular rib foam, $I=mt^4/12$ and $\mu=\rho_s mt^2$ for rectangular rib foam, the cut-off frequency ν_{cut} (Hz) of open-cell foams is obtained as

$$\nu_{cut} = f(\theta)\,\frac{1}{2\pi}\,\frac{L}{(d_{cell})^2}\sqrt{\frac{E}{\rho}} \tag{18}$$

(circular cross section),

or $$\nu_{cut} = f(\theta)\,\frac{1}{\pi}\sqrt{\frac{2}{3(m^2+1)}}\,\frac{L}{(d_{cell})^2}\sqrt{\frac{E}{\rho}} \tag{19}$$

(rectangular cross section),

in which the foam cell diameter $d_{cell} = 2R$, and E is Young's modulus of the foam. We remark that Eq. 10 gives the foam stiffness based on *bending* of rib elements. Since curved rib elements have bending rigidity nearly identical to that of straight rib elements, the same relation can be used.

For the vibration of a partial ring normal to its own plane, the cut-off frequency of open-cell foams is obtained to be

$$\nu_{cut} = f(\theta,C)\,\frac{1}{2\pi}\,\frac{L}{(d_{cell})^2}\sqrt{\frac{E}{\rho}} \tag{20}$$

(circular cross section),

or $$\nu_{cut} = f(\theta,C)\,\frac{1}{\pi}\sqrt{\frac{1}{3}}\,\frac{L}{(d_{cell})^2}\sqrt{\frac{E}{\rho}} \tag{21}$$

(square cross section)

in which $f(\theta,C)$ is a coefficient similar to $f(\theta)$ in Eq. (17) but with additional considerations on the bending stiffness and torsional stiffness of the ring. Only the cut-off frequency of circular and square cross section rib foams is derived as a result of the complication caused by C in Eq. (21).

The convolutedness parameter Ξ of a partial ring of sector angle θ from 180° to 360° is defined as

$$\Xi = \frac{L}{2R}$$

$$= \theta \frac{\pi}{360} \qquad (22)$$

with values from 1.571 to 3.142 for a rib modelled as a partial ring.

2.3. Foam rib modeled as a helical spring

Here the foam rib is so curved that it loops upon itself, and is describable as a helical spring. The governing equation of a helical spring vibrating in the axial direction has been obtained as [14]

$$\frac{d^2u}{dx^2} = \frac{1}{2} \frac{(D_m)^2}{r^2} \frac{\rho_s}{G_s} \frac{d^2u}{dt^2} \qquad (23)$$

(circular cross section),

or $$\frac{d^2u}{dx^2} = \frac{2.45m}{m-0.56} \frac{(D_m)^2}{t^2} \frac{\rho_s}{G_s} \frac{d^2u}{dt^2} \qquad (24)$$

(rectangular cross section),

in which x is the axial direction of the spring, t is the time, u is the deflection in the x direction, D_m is the mean diameter of the coil, and G_s is the shear modulus of the spring material.

Solving Eqs. (23) and (24) with proper boundary conditions applied, the natural frequency ν_{spring} (in Hz) of a helical spring vibrating in the axial direction with clamped-clamped ends or free-free ends is derived as

$$\nu_{spring} = \frac{1}{\sqrt{2}\pi} \frac{1}{n_s} \frac{r}{(D_m)^2} \sqrt{\frac{G_s}{\rho_s}} \qquad (25)$$

(circular cross section),

or $$\nu_{spring} = \frac{1}{2} \sqrt{\frac{m-0.56}{2.45m}} \frac{1}{n_s} \frac{t}{(D_m)^2} \sqrt{\frac{G_s}{\rho_s}} \qquad (26)$$

(rectangular cross section),

in which n_s is the number of the coils of the spring and G_s is the shear modulus of the solid comprising the spring elements which represent foam ribs. The natural frequency of a helical spring with circular coils given by Eq. (25) is identical to that published elsewhere [10].

Substituting Eqs. (10) and (11) into Eqs. (25) and (26) with $G/G_s = E/E_s$ and $d_{cell} = D_m$ applied, the cut-off frequency ν_{cut} (Hz) of open-cell foams with ribs modeled as helical springs is obtained to be

$$\nu_{cut} = 0.113 \, \frac{1}{n_s} \, \frac{L}{(d_{cell})^2} \sqrt{\frac{G}{\rho}} \tag{27}$$

(circular cross section),

or $$\nu_{cut} = 0.5 \sqrt{\frac{2(m-0.56)}{2.45m(m^2+1)}} \, \frac{1}{n_s} \, \frac{L}{(d_{cell})^2} \sqrt{\frac{G}{\rho}} \tag{28}$$

(rectangular cross section),

in which G is the shear modulus of the foam and ρ is its density.

The convolutedness parameter Ξ of a helical spring of n_s turns is defined as

$$\begin{aligned} \Xi &= \frac{n_s D_m \pi}{D_m} \\ &= n_s \pi \end{aligned} \tag{29}$$

It is possible in principle to have a helical foam rib with many turns, but such ribs have not been practically realized.

As given by Eqs. (16), (22) and (29), the convolutedness Ξ of three different vibrational members which are used to model the foam ribs, are defined consistently. The convolutedness Ξ can also be considered as the ratio of the arc length to the maximum straight distance between any two points along such a member. For a curved beam of sector angle θ between 0° and 90°, the value of the convolutedness parameter Ξ is 1 as a straight beam and increases to 1.11 as a curved beam of θ = 90°. For a partial ring of sector angle θ between 180° and 360° and a coil spring, the value of the convolutedness Ξ is defined as the ratio of the ring length to the ring diameter and increases from 1.571 or $\pi/2$ as a half ring to 3.142 or π as a complete coil, and to $n_s\pi$ as a coil spring of coil number (number of turns) n_s. However, the cut-off frequency ν_{cut} of a partial ring of θ

from 90° to 180° or Ξ from 1.11 to 1.571 is not well obtained yet as a result of a numerical technique problem.

3. Materials

Results of free-free resonance experiments in torsion conducted at room temperature were published earlier [4]. Two types of viscoelastic foams, polyester foam and Scott industrial foam, were tested. The polyester foam is partly open cell and partly closed cell and Scott industrial foam is all open cell. Partial closed cells are considered to behave as open cells since substantial material must be in the cell walls of closed cell foams to result in significant differences in the mechanical properties according to Gibson and Ashby [12]. The properties which are required for cut-off frequency predictions of these two conventional viscoelastic foams are listed in Table 1.

4. Results and discussion

The cut-off frequencies of conventional viscoelastic foams were obtained experimentally as 2500 Hz and 1000 Hz for polyester foam and Scott industrial foam respectively, as indicated earlier [4]. The experimental results, as well as predictions obtained with foam ribs modeled as curved beams, partial rings and helical springs, are shown in Figs. 1 and 2. One might expect the cut-off frequency to behave as a continuous function of the convolutedness parameter. However different geometrical configurations are considered as approximations of the cell rib for each part of the range. Therefore different functional relations result.

Observe that both types of conventional foam exhibit observed cut-off frequencies considerably lower than the values predicted assuming straight ribs. This comparison is in contrast to one made earlier [4]. The reason is that a vibration formula for straight bars, taken from Ferry [15] was later found to be incorrect by a factor of $(2\pi)^2$; the correct relations are used in the present work. Conventional foams, therefore, behave as if the ribs were convoluted rather than straight. In the actual foams the ribs are slightly curved in the conventional foams, and sharply curved into loops (not springs) in the re-entrant foams. The difference is attributed to concentrations of mass at the junctions of the cell ribs. Such concentrations were found to be difficult to incorporate into further refinements of the modelling. Another possible structural feature not yet accounted for includes the presence of plate or membrane elements in the polyester foam. Gibson and Ashby [12] have considered the effect of such elements in the static behavior of foams. If membrane elements are thin enough, they may resonate at a lower frequency than rib elements. One may also consider the role of interaction between adjacent cells. Multi-cell theories have been considered in detail in solid state physics for the analysis of vibration in crystal lattices [16]. In such theories, it is the size of the unit cell, not the size of ensembles of cells, which governs the dispersion and cut-off frequency for acoustic waves. Therefore understanding the single cell resonant behavior of foams is sufficient unless they contain structure on a larger scale.

As for viscoelasticity, the foam ribs of a polymer foam can be expected to exhibit mechanical damping as quantified by the loss tangent tan δ. The tan δ of the cell rib material will affect the breadth of the rib resonance but not to first order the rib resonance frequency. The present analysis deals with the cut-off frequency for the foam, which depends on resonance of the ribs. Viscoelasticity will not to first order affect the conclusions.

As for the negative Poisson's ratio foams, the structure is clearly more convoluted [5] than that of conventional foams, so that lower cut-off frequencies are expected and they are observed for negative Poisson's ratio foams. There is considerable freedom the structure of negative Poisson's ratio materials: the initial density, permanent volumetric compression, and rib material properties are all independent variables. The optimal combination which would result in the most favorable acoustic behavior is as yet not known. Even so, negative Poisson's ratio foams studies thus far exhibit higher sound absorption and lower sound reflection characteristics compared with foams of conventional structure.

5. Conclusions

1. Cut off frequencies are predicted based on modeling of micro-vibration of cell ribs which may be straight, curved, or helical.
2. The predicted cut-off frequency decreases as the ribs become more convoluted.

TABLE 1

Foam material properties

	Young's modulus E (KPa)	Shear modulus G (KPa)	Mass density ρ (g/cm^3)	Cell size d_{cell} (mm)	Rib length L (mm)
Conventional Polyester foam	211	81	0.0310	0.5	0.4
Re-entrant Polyester foam	211	81	0.0860	0.35	0.4
Conventional Scott industrial foam	86	33	0.0304	2.5	1.5
Re-entrant Scott industrial foam	75	29	0.108	1.6	1.5

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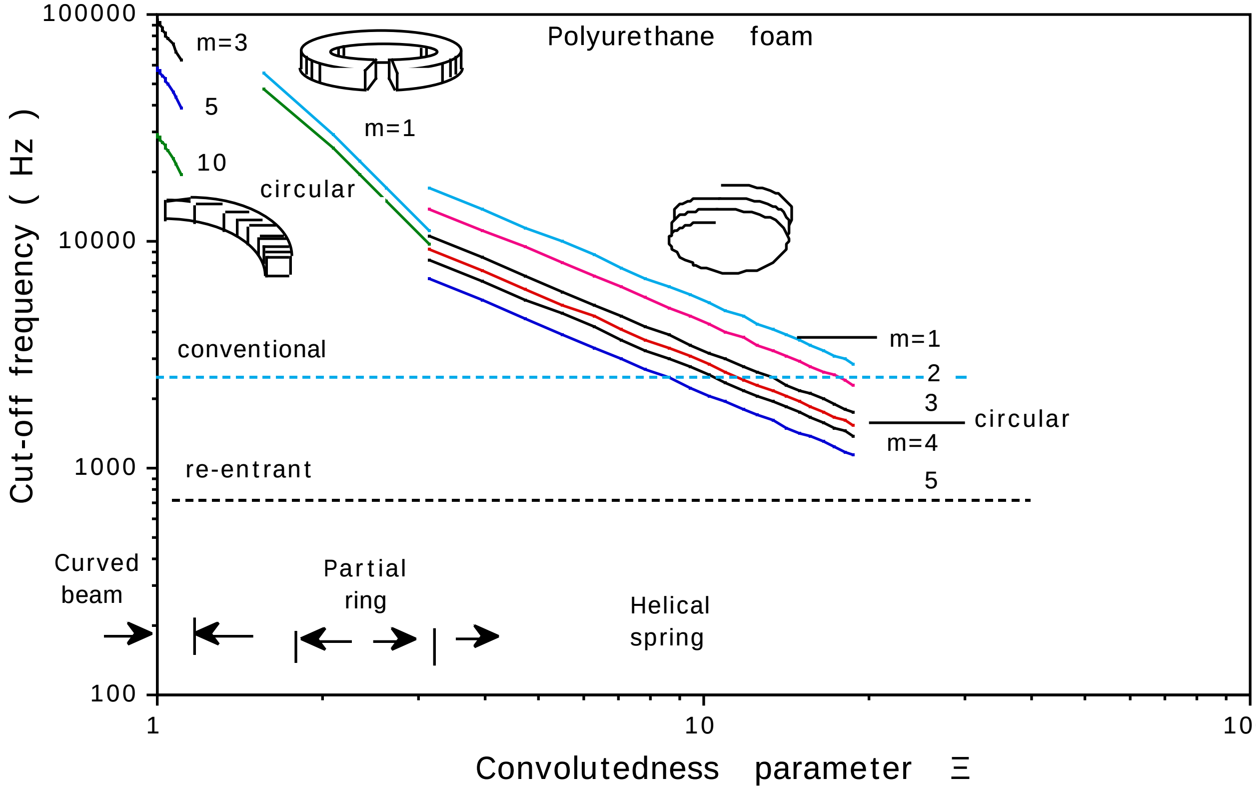
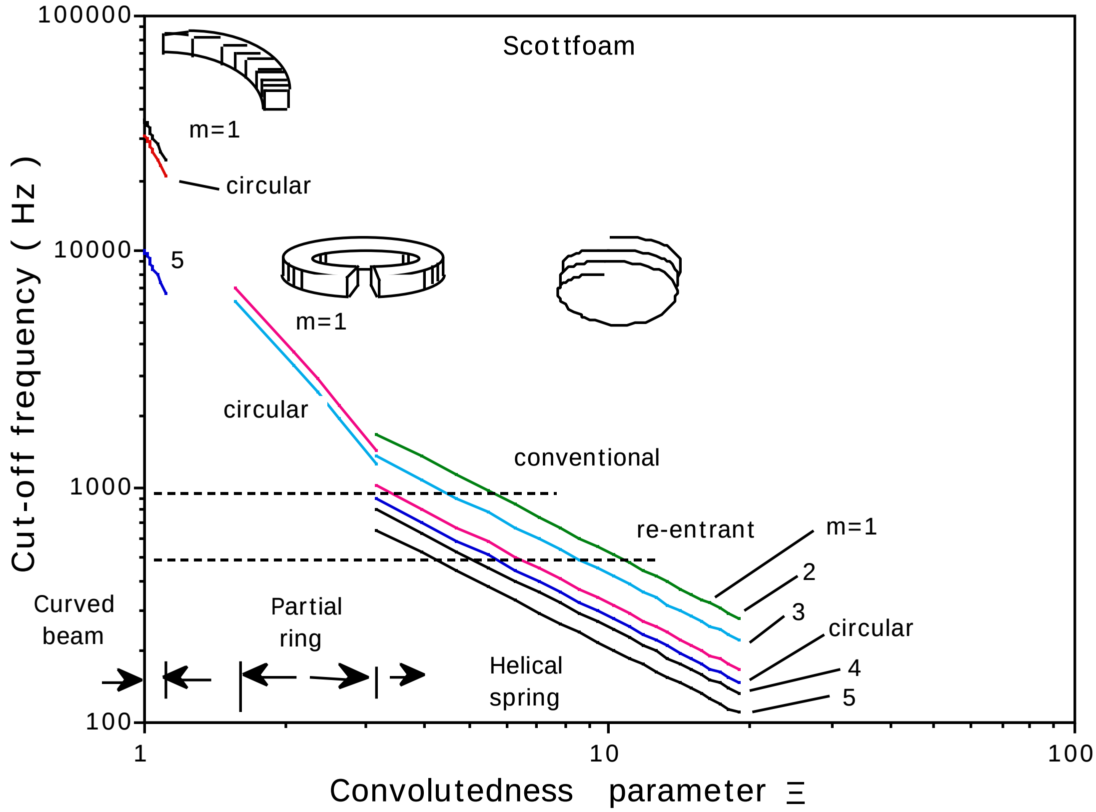


1. Modeled cut-off frequency vs convolutedness parameter Ξ of ribs, polyester foam.

Assumed input properties are those of a polymer foam studied in the laboratory, vibration normal to plane, clamped-clamped ends or free-free ends.

As indicated, circular cross section ribs, and rectangular cross section ribs, for various cross section shapes denoted by m.



2. Modeled cut-off frequency vs convolutedness parameter Ξ of ribs, Scott industrial foam. Assumed input properties are those of a Scott industrial foam studied in the laboratory, vibration normal to plane, clamped-clamped ends or free-free ends.

As indicated, circular cross section ribs, and rectangular cross section ribs, for various cross section shapes denoted by m.