
**Abstract**

Micro-deformation studies of conventional and re-entrant negative Poisson's ratio copper foams were conducted holographically. Inhomogeneous, non-affine deformation was observed holographically in both foam materials. The re-entrant material with a permanent volumetric compression ratio 2.2, and a negative Poisson's ratio -0.8 exhibited a substantially greater non-affine deformation than the conventional material, in contrast to foam with compression ratio 3.0 examined earlier.

**1. Introduction**

Conventional foams, like other ordinary materials, exhibit a positive Poisson's ratio, that is, they become smaller in cross-section when stretched and larger when compressed. Recently, the invention of negative Poisson's ratio foams was reported [1,2,3]. Foam materials based on metal and several polymers were transformed so that their cellular architecture became re-entrant, i.e. with inwardly protruding cell ribs. Foams with re-entrant structures exhibited negative Poisson's ratios as well as greater resilience than conventional foams.

An experimental study by holographic interferometry was reported of the Young's moduli, Poisson's ratios, yield strengths and characteristic lengths associated with inhomogeneous deformation of the conventional and negative Poisson's ratio metallic foams [4]. The Young's modulus and yield strength of the conventional copper foam were comparable to those predicted by microstructural modelling on the basis of cellular rib bending. The re-entrant copper foam exhibited a negative Poisson's ratio as indicated by the elliptic contour fringes on the specimen surface in the bending tests. Inhomogeneous, non-affine deformation was observed holographically in both foam materials.

The present study applies double-exposure holographic interferometry to examine micro-deformation of re-entrant copper foam with a negative Poisson's ratio of -0.8.
2. Material and method

Bending experiments were conducted upon re-entrant copper foam specimens at room temperature. The re-entrant structure was transformed as that a maximum value of negative Poisson's ratio -0.8 at a strain level of 0.1% as determined earlier [5] by shadow moiré. The conventional copper foam used was open cell with density 0.715 g cm\(^{-3}\), solid volume fraction 0.08, and average cell size 0.4 mm. It is worth noting that the density of copper foam was not uniformly distributed. The density measured on different portions inside a foam block can vary from 0.08 to 0.1 [5]. The copper foam was transformed into re-entrant structure by successive applications of small increments of plastic deformation in three orthogonal directions, as described earlier [1,2,3]. The density of the re-entrant copper foam tested here was 1.57 g cm\(^{-3}\) with a permanent volumetric compression ratio of approximately 2.2, and solid volume fraction 0.22. The dimensions of the re-entrant copper foam specimen was 5.7 mm by 10.8 mm in cross section and 35.5 mm in length. For the purpose of comparison, a re-entrant foam specimen was annealed at 500°C before being tested. The anneal process did not result in measurable change in the foam dimensions. The dimensions of the annealed re-entrant copper foam specimen was 7.75 mm by 7.9 mm in cross section and 36 mm in length. The specimens were machine finished to obtain the desired surface smoothness.

The experimental study of the re-entrant copper foams by holographic interferometry is as described earlier [4]. The specimen was deformed by a nearly pure bending moment. The zero-order fringe method (ZF) [6] was used to obtain the specimen's Young's moduli and Poisson's ratios from the fringe pattern on the strained surface of the specimen. The non-affine deformation of the re-entrant foam was also studied by the zero-order fringe method. The inhomogeneous deformation of the non-affine type was visualized as an increase in the bumpiness of the fringes in response to the increasing loads. The results were compared with those obtained earlier [4].

3. Results and Discussion

The results of the Young's moduli and Poisson's ratios are listed in Table 1, in comparison with the previous results for the re-entrant copper foam with a permanent volumetric compression ratio of approximately three [4]. Each Young's modulus was determined at two strain levels. The Young's modulus remained approximately constant when the strain level was increased. For the non-annealed re-entrant copper foam, the Young's modulus was found to be 390 MPa at a macroscopic strain of 1.4 x 10\(^{-4}\) and 1.7 x 10\(^{-4}\) and 200 MPa at a macroscopic strain of 1.6 x 10\(^{-4}\) and 3.0 x 10\(^{-4}\), for observation of the strained surface of width 10.8 mm and 5.7 mm, respectively. For the annealed re-
entrant copper foam, the Young's modulus was obtained to be 510 MPa at a macroscopic strain of 9.5 x 10^{-5} and 1.8 x 10^{-4}; and 245 MPa at a macroscopic strain of 9.7 x 10^{-5} and 1.8 x 10^{-4}, on the strained surface of width 7.9 mm and 7.75 mm, respectively. In the previous results [4] for the re-entrant copper foam with a permanent volumetric compression ratio of approximately 3, the Young's modulus was obtained to be 480 MPa at a macroscopic strain of 6.7 x 10^{-4}.

The Young's moduli obtained from bending in two orthogonal directions differed by a factor of two. Anisotropy will not give rise to such an effect since a bending experiment in any transverse direction discloses the Young's modulus for strain along the beam axis even if the beam is anisotropic. Anisotropy could arise from different permanent compression in the three orthogonal directions, however care was taken to avoid that. Nonuniformity in the permanent compression process could give rise to such results.

The fringe pattern obtained for the current studies of negative Poisson's ratio foam of volumetric compression ratio 2.2 was observed to be bumpier than that obtained previously [4] for denser negative Poisson's ratio foam. The comparison of the clarity of the fringes of the re-entrant foams is shown in Figure 1. The non-affine (inhomogeneous) deformation disclosed by these experiments is an average value perpendicular to the specimen surface. The maximum strain level \( \varepsilon \) for the minimum observable clarity of the fringe pattern was \( \varepsilon = 2.5 \times 10^{-4} \), well below 9.2 x 10^{-4} as obtained previously [4] for re-entrant foam of a higher permanent volumetric compression (3.0). The Poisson's ratio for both non-annealed and annealed re-entrant copper foams could not be determined in this experiment since the fringes could only be recognized at a low macroscopic strain level. At low strain, only a small part of the elliptic fringes was visible on the surface and therefore the Poisson's ratio was not obtainable. This observation is in contrast to results of an earlier study by the shadow moiré method, in which the Poisson's ratio of this type of re-entrant copper foam was determined to be -0.8 [5], at a strain of 10^{-3}. A second shadow moiré experiment was performed following the holographic study and the results were found to be repeatable. Both shadow moiré and holography as used in these experiments are sensitive to the component of displacement along the line of sight, perpendicular to the specimen surface. The difference is that in these studies, holography has a resolution of 0.316 \( \mu \)m per fringe, while shadow moiré has a resolution of 18 \( \mu \)m per fringe. Therefore the moiré experiments were conducted at higher strains (10^{-3}) than the holography, though well below the yield strain of more than 6 x 10^{-3} [5] of this material. The difficulty in measuring Poisson's ratio by holography for this re-entrant foam is attributed to nonlinear behavior of the material at small strain well below the yield strain. Indeed, ref. [5] discloses a cusp-like behavior of Poisson's ratio versus strain for copper foam, down to strains as small as 10^{-3}. A likely
physical mechanism for such nonlinearity is the frictional contact between ribs in the re-entrant foam structure; see the micrographs shown in References [3,5].

Quantitative measures of inhomogeneous deformation may be defined as follows: the ratio $\Gamma$ of micro-strain to macro-strain $\varepsilon_{\text{macro}}$, $\Gamma = (\delta/d_{\text{cell}})/\varepsilon_{\text{macro}}$; and a micro-deformation characteristic length $L_m = \delta/\varepsilon_{\text{macro}}$ in which $\delta$ is the inhomogeneous micro-deformation displacement and $d_{\text{cell}}$ is the cell size. In the present zero-order fringe experiments the fringes become totally fragmented when the micro-deformation over one cell is one fringe or half a wavelength of red helium neon laser light, so $\delta = 0.3 \mu m$. The cell size is $0.4 \text{ mm}$ for conventional foam and $0.4 \text{ mm}/(2.2)^{1/3}$ for the present re-entrant foam. Consequently, for the conventional copper foam [4], $\Gamma_{\text{avg},\perp} = 2.1$, and $L_m = 0.86 \text{ mm}$; for the previous re-entrant foam of permanent compression ratio 3, $\Gamma_{\text{avg},\perp} = 1.3$, $L_m = 0.38 \text{ mm}$; for the present re-entrant foam, $\Gamma_{\text{avg},\perp} = 3.9$, $L_m = 1.2 \text{ mm}$. So in the present re-entrant foam, both measures of the degree of inhomogeneous deformation are greater than in the case of conventional foam or a previous re-entrant foam. The difference is attributed to contact between ribs of the re-entrant foam; see the micrographs shown in References [3,5]. At higher volumetric compression ratios, there are more such contacts, impeding the micro-movements which give rise to the negative Poisson's ratio and to the non-affine (inhomogeneous) deformation.

The inhomogeneous, non-affine micro-deformation observed in the foam has several interesting implications as discussed in more detail in an earlier communication [4]. Micro-deformation is evidence of generalized continuum behavior in the material [7]. This kind of generalized continuum effect is to be distinguished from Cosserat elasticity, a generalized continuum theory in which the points of the continuum, or unit cell nodes in the actual structure, can rotate with respect to each other. Microstructure [7] or micromorphic elasticity incorporates both the micro-rotation of Cosserat elasticity and a micro-deformation as well. Cosserat elasticity has been explored experimentally in several foams and fibrous materials as reviewed in [8]; results for re-entrant foam were reported as well. Local micro-rotation degrees of freedom of Cosserat elastic materials are intimately connected with a toughening mechanism governed by redistribution of stress around cracks and holes [8]. Stress concentration factors for holes and cracks are lower in Cosserat solids than in classical solids. The effect of the micro-deformation degrees of freedom are not as well understood. However it is known theoretically that stress concentration factors for spherical cavities [9] can be higher in the presence of micromorphic behavior. Moreover, in a special type of micromorphic solid, allowing only micro-dilatations but no micro-rotations, the predicted stress concentration factor for a hole is greater than the classical prediction [10]. To summarize, the current study has demonstrated a larger inhomogeneous
deformation, hence micro-deformation in the generalized continuum sense, in a particular kind of re-entrant foam than in conventional foam. Cosserat elasticity in re-entrant foam was demonstrated in a previous study. Which effect will be most important in governing stress concentration factors around holes remains to be determined.

4. Conclusions

1. Negative Poisson's ratio copper foam with a permanent volumetric compression ratio of 2.2 exhibits a greater non-affine (inhomogeneous) deformation than either conventional foam or negative Poisson's ratio foam with a volumetric compression ratio of 3.

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References
1. The clarity of the fringe pattern versus the macroscopic strain, bending test. _ : Conventional copper foam, density 0.795 g cm$^{-3}$, Poisson's ratio 0.25, specimen size 9 mm by 15 mm by 34 mm, _ : Re-entrant copper foam, compression ratio 3.0, Poisson's ratio -0.11, specimen size 6 mm by 6.5 mm by 26 mm, ♦ : Re-entrant copper foam, compression ratio 2.2, Poisson's ratio -0.8 from shadow moiré [5], specimen size 5.7 mm by 10.8 mm by 35.5 mm.
Table 1. Specimen dimensions and properties

Width of surface at maximum strain is last dimension in 'specimen size'.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Specimen size (mm)</th>
<th>Permanent compression ratio</th>
<th>Poisson's ratio</th>
<th>Young's modulus</th>
<th>Inhomogeneous deformation</th>
</tr>
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<tbody>
<tr>
<td>Re-entrant</td>
<td>26 x 6 x 6.5</td>
<td>3.0</td>
<td>-0.11</td>
<td>480 MPa</td>
<td>1.3</td>
</tr>
<tr>
<td>Re-entrant</td>
<td>35.5 x 5.7 x 10.8</td>
<td>2.2</td>
<td>-0.8 [5]</td>
<td>390 MPa</td>
<td>3.9</td>
</tr>
<tr>
<td>copper foam</td>
<td>35.5 x 10.8 x 5.7</td>
<td></td>
<td></td>
<td>200 MPa</td>
<td></td>
</tr>
<tr>
<td>Re-entrant</td>
<td>36 x 7.75 x 7.9</td>
<td>2.2</td>
<td>N/A</td>
<td>510 MPa</td>
<td>3.9</td>
</tr>
<tr>
<td>copper foam</td>
<td>36 x 7.9 x 7.75</td>
<td></td>
<td></td>
<td>245 MPa</td>
<td></td>
</tr>
<tr>
<td>annealed</td>
<td>36 x 7.75 x 7.9</td>
<td>2.2</td>
<td>N/A</td>
<td>510 MPa</td>
<td>3.9</td>
</tr>
<tr>
<td>at 500°C</td>
<td>36 x 7.9 x 7.75</td>
<td></td>
<td></td>
<td>245 MPa</td>
<td></td>
</tr>
</tbody>
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