Comparison of FRF and Modal Methods for Combining Experimental and Analytical Substructures

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Outline

- Motivation
- Modal Coupling vs. FRF Coupling
- Component Mode Synthesis Theory
- Methods for connecting subsystems
  - Connection Point Method (CPT)
  - Modal Constraint for Fixture and Subsystem (MCFS)
- Experimental Results
  - Rigid Fixture Model with CPT constraint
  - Elastic Fixture Model with CPT constraint
  - Elastic Fixture with MCFS constraint
- Conclusions
Subcomponents are often designed by a number of independent groups that do not have the information or the resources to model the macro system.

In other applications some particular components may be too difficult to model analytically with the required precision.
Modal vs. FRF Coupling

- Two distinct approaches to substructure coupling exist:
  - Component Mode Synthesis or Modal Coupling:
    - Modal models for substructures combined to find an approximate modal model for the total system.
    - Requires modal parameter estimation to reduce measurements to a modal system model.
    - Measurements must be sufficient to obtain reasonably accurate estimates of each important modal frequency and mode shape at the connection points.
    - Results can sometimes be understood by appealing to well-established Ritz Theory.
    - Disadvantage: It is not always easy to identify an adequate modal model from experimental measurements.
  - FRF Based Admittance or Impedance Coupling:
    - Response measurements (FRF matrices) for substructures combined to find FRFs for total system.
    - Can be performed on raw FRFs (even if modal parameter estimation is not feasible.)
      - However, all connection point FRFs must be measured if MPE is not employed.
        - $6N_c \times 6N_c$ set of FRFs!
      - Numerical ill-conditioning may present a formidable challenge.
      - Modal parameter estimation is very desirable to reduce measurement errors for lightly damped systems. [Imregun, Robb, Ewins IMAC-1987]
Component Mode Synthesis

- Given a set of modal parameters, one has a set of equations of motion (EOM) for a substructure:

\[
[I] \{\ddot{\eta}\} + [\omega_r^2] \{\eta\} = [\Phi]^T \{F\} \\
\{y\} = [\Phi] \{\eta\}
\]

- \{y\} is a vector of physical coordinates
- \{\eta\} are modal coordinates
- \([\omega_r^2]\) is a diagonal matrix of natural frequencies squared
- \([\Phi]\) is a matrix of mass-normalized mode vectors.

- Consider two independent structures A and B:

- Their EOM are....
Component Mode Synthesis (2)

\[
\begin{bmatrix}
[I]_A & 0 \\
0 & [I]_B
\end{bmatrix}
\begin{bmatrix}
\{\ddot{\eta}\}_A \\
\{\ddot{\eta}\}_B
\end{bmatrix}
+ \begin{bmatrix}
[\omega^2_T]_A & 0 \\
0 & [\omega^2_T]_B
\end{bmatrix}
\begin{bmatrix}
\{\eta\}_A \\
\{\eta\}_B
\end{bmatrix}
= \begin{bmatrix}
[\Phi]_A^T\{F\} \\
[\Phi]_B^T\{F\}
\end{bmatrix}
\]

- Connect them by enforcing linear constraints:
  \((y_c)_A = (y_c)_B\)
- All DOF can be expressed in terms of an unconstrained set of generalized coordinates as
  \[
  \begin{bmatrix}
  \{\eta\}_A \\
  \{\eta\}_B
  \end{bmatrix}
  = [P] \begin{bmatrix}
  -[\hat{a}_c]^{-1} \\
  [I]
  \end{bmatrix}
  \{\eta\}_u = [B]\{\eta\}_u
  \]
- The equations of motion in unconstrained coordinates are then:
  \[
  [\hat{M}]\{\ddot{\eta}\}_u + [\hat{K}]\{\eta\}_u = \{Q\}
  \]
  \[
  [\hat{M}] = [B]^T
  \begin{bmatrix}
  [I]_A & 0 \\
  0 & [I]_B
  \end{bmatrix}
  \]
  \[
  [\hat{K}] = [B]^T
  \begin{bmatrix}
  [\omega^2_T]_A & 0 \\
  0 & [\omega^2_T]_B
  \end{bmatrix}
  \]

Paper includes a method that chooses constrained DOF so that this matrix is always invertible if such a choice exists.
Component Mode Synthesis (3)

- One can now find the modes of the combined system, construct FRFs, etc… using these EOM.
- The mode shapes of the combined system are found in the physical coordinates of A and B.
  - The combined system mode shapes are linear combinations of the mode shapes of the subcomponents A and B.
  - One must assure that enough modes of both A and B are present so that these are a good approximation for the true modes of the combined system.
Objective: Join an experimental model of beam B to an analytical model for beam D at point $y_c$.

- Measurements are taken on system C consisting of beam B with fixture A attached.
- Analytical model (Euler-Bernoulli) for fixture A removed from system C resulting in an experimental model for beam B.
  - Analytical model for fixture A
- Beam B is then combined with an analytical model (tuned Euler-Bernoulli) for beam D to find the combined system Beam E.
Careful tests used to estimate the modal parameters of the C system at a number of points on fixture A.
Connection Methods – CPT and MCFS

- One cannot measure the response at the connection point directly, so it must be estimated from other measurements.
  - CPT Method: Connection point responses for the experimental system are estimated using a modal filter and constrained to the analytical fixture A and beam D:
    \[
    \begin{bmatrix}
    \{y^C\}_m \\
    \{y^C\}_c
    \end{bmatrix}
    \approx
    \begin{bmatrix}
    \Phi^A_m \\
    \Phi^A_c
    \end{bmatrix}
    \{\eta^C\}
    \rightarrow
    \{y^C\}_c = \left[\Phi^A_c [\Phi^A_m]^\dagger\right] \{y^C\}_m
    \]
  - MCFS Method: Constrain the modal DOF of the Fixture model to their approximation on C: (MCFS stands for Modal Constraint for Fixture and Subsystem)
    \[
    \{\eta^A\} = [\Phi^A_m]^\dagger \{y^A\}_m
    \]
    \[
    \{\eta^C\} = [\Phi^A_m]^\dagger \{y^C\}_m
    \rightarrow
    \{\eta^A\} = \{\eta^C\}
    \]
CMS can be very sensitive to errors when removing a substructure from a system.

In the problem considered here the fixture response is dominated by (4) modes.

Requiring equal 2D motion at the connection point enforces only (3) constraints.

- \(4\text{ DOF} \times 2\text{ Systems} \quad - \quad 3\text{ Constraints} = 5\text{ Remaining DOF}
- Two elastic modes remain in the system. One would hope that these have no effect on the response.
- It might be preferable to remove these extra modes by adding constraints rather than simply hope that the fixture model is accurate enough so that they cancel.
Cases Considered

- **Case 1:** CPT: Models for A, C and D joined at the connection point.
  - Case 1a: Rigid Fixture Model
  - Case 1b: Elastic Fixture Model

- **Case 2:** MCFS: Models for A and C joined using MCFS method.
  - Elastic Fixture Model
Case 1a: Rigid A, CPT

- Excellent results obtained in the lateral direction (Bending Modes).
- Both the CMS and FRF based Admittance procedures agree very well with the analytical model.
- The CMS result is slightly contaminated by the axial modes at 2400 and 5200 Hz.
Case 1a: Rigid A, CPT

- Admittance and CMS both under-predict the natural frequencies of the axial modes by 10% or more.
  - These errors are larger than one would expect due to modal truncation alone.
  - Rigid fixture model is not adequate.
- Both also over-predict the axial motion in the bending modes resulting in contamination at the bending natural frequencies.
  - Possibly due to small curve fitting errors or cross axis sensitivity.
Case 1b: Flexible A, CPT

- Both Admittance and CMS accurately predict the first axial frequency with the flexible fixture model.
- Both methods more severely under-predict the second axial mode.
- Both predict a spurious zero near 5500 Hz.
- The Lateral FRFs were similar to those shown previously.
Case 2: Flexible A, MCFS Method

- The CMS method predicts the axial FRF very accurately when the Modal Constraint for Fixture and Subsystem (MCFS) is used.
Physically Realizable Models

- The E system models obtained were not completely physical.
  - Some of the E system models had complex natural frequencies
  - The E system mass matrix was not always positive definite.
- This is to be expected since an approximation to the stiffness of the fixture has been removed.
- When combining structures, the spurious modes all appear at high frequencies, yet they can appear at lower frequencies when removing a substructure.
  - In these cases, the spurious natural frequencies are always near the extremes of the frequency band.
  - Admittance results also show non-physicality (negative eigenvalues of drive point FRFs at some frequencies).

<table>
<thead>
<tr>
<th>Case</th>
<th>Complex $f_n$ (Hz)</th>
<th>eig(M) &lt; 0</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Case 1a</strong></td>
<td>0 + i*8.93e-5</td>
<td>-0.011</td>
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<tr>
<td>Rigid Fixture, Con. Pt.</td>
<td>0 + i*54100</td>
<td></td>
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<tr>
<td><strong>Case 1b</strong></td>
<td>0 + i*1.36e-4</td>
<td>-1</td>
</tr>
<tr>
<td>Flexible Fixture, Con. Pt.</td>
<td>8951 – i*2450</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8951 + i*2450</td>
<td></td>
</tr>
<tr>
<td><strong>Case 2</strong></td>
<td>0 + i*2.28e-4</td>
<td>-0.086</td>
</tr>
<tr>
<td>Flexible Fixture, MCFS</td>
<td>13050 – i*4285</td>
<td></td>
</tr>
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It could be worse!

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Conclusions

- **Lateral (bending direction)**
  - Both Component Mode Synthesis (CMS) and FRF based Substructuring (Admittance) can be used to accurately predict the lateral modes of this Experimental-Analytical system.

- **Axial direction:**
  - It was necessary to account for the elasticity of the fixture to obtain accurate estimates of the axial modes.
  - When doing so, the Modal Constraint for Fixture and Substructure (MCFS) method improved the accuracy of the predictions when compared to the connection point method (CPT).
  - MCFS allows one a new level of freedom when designing test fixtures for these types of analyses.
  - It would be helpful to have a method of removing test fixtures that assures that a physically meaningful model is obtained.