Negative Poisson's ratio in 2D Voronoi cellular solids by biaxial compression: a numerical study

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Abstract A 2D (two dimensional) random cellular solid model was built using FEM(finite element method) based on a modified Voronoi tessellation technique. A sequence of permanent biaxial compression deformations was applied on the model to obtain a series of re-entrant random cellular solid structures with different area compression ratios. The Poisson's ratio and energy absorption capacity of cellular solid models with different initial relative densities (0.032 and 0.039) were studied at different area compression ratios. The results showed that the Poisson's ratio first decreased and then increased with increasing compression strain. A minimum Poisson's ratio of approximately -0.38 was achieved with an appropriate compression strain. An empirical fitting rule was established which can best fit the 2D simulation to 3D experimental results for foams. The cellular solids with minimum negative Poisson's ratio can exhibit the highest energy absorption capacity. Furthermore, mechanical properties of the random cellular solid model were compared with 2D regular honeycomb models with both concave and convex shaped cells. Results showed that the energy absorption capacity of the three models increased with an increasing dynamic compression velocity. The random foam model exhibited the highest increase rate in energy absorption capacity with the increasing compression velocity.

Keywords: Re-entrant; cellular solid; Negative Poisson’s ratio; Voronoi; Energy absorption; Compression

Introduction

Cellular solids, including engineering honeycombs and foams, are widely used in many structural applications due to the low weight and high energy absorption capability. The mechanical behaviors of cellular solids depend on the properties of their parent material, the relative density and their microstructure, as presented by Gibson and Ashby [1]. Honeycombs can exhibit a negative Poisson's ratio if the cells in plane have an inverted bow-tie shape [2, 3]. Honeycomb is always anisotropic: the out of plane stiffness greatly exceeds the in plane stiffness [1, 4, 5]. Two dimensional isotropy is possible; the Poisson's ratio is +1 for regular hexagons and can be tuned to -1 for appropriate geometry. Other 2D negative Poisson's ratio structures include rigid hexagons [6] and hinged squares [7]. A negative Poisson's ratio is possible in a 2D hierarchical elastically isotropic composite [8]; the Poisson's ratio tends to -1 as the contrast between the constituent moduli increases.

Isotropic(3D) foam materials with negative Poisson's ratios have been fabricated and characterized by the authors [9, 10]. For polymer foams [9], the material was triaxially compressed, heated above the softening point, and cooled to ambient temperature. For ductile metal foams [9], the material was plastically compressed in small increments to attain a triaxial compression strain of an appropriate magnitude. Foam materials with cells of re-entrant shape have been made with Poisson's ratio as small as -0.8. Negative Poisson's ratio gives rise to a predicted increase in some material properties such as flexural rigidity and plane strain fracture toughness [11]. Re-entrant transformation of copper foam resulted in an increase in the apparent indentation resistance, [12], consistent with theory [13].

Analytical study provides closed form solutions for regular (2D) honeycomb with slender rib elements [9]. 3D microstructures such as foams and lattices have also been anayzed; for slender ribs the density dependence of properties has been obtained by elementary analysis. Foam or lattice with regular tetrakadecahedron cell structure has been analyzed via a full 3D approach [14]. As for negative Poisson's ratio foam, an elementary analysis of a single cell was sufficient to capture some aspects of the behavior [15]. When irregularity of the cells is incorporated or effects of interactions among many cells is considered, exact analysis becomes impractical and use of finite element method is appropriate. Voronoi models in conjunction with the finite element method are most widely used to study random foam materials. 2D Voronoi models were originally developed using a finite element method by Gibson, et. al [16, 17]. Chen et al. [18] studied the effects of morphological imperfection on the yielding of cellular solids using 2D Voronoi random models. Zhu et al. [19] built a finite element model of random Voronoi honeycombs, and studied the cell irregularity effects on the elastic properties of 2D random foams.

There are also many studies on 3D Voronoi models using finite element method, but most of them were limited to the foams' elastic behaviours due to the computational costs [20-23]. In addition, most of these studies were performed on conventional foams rather than re-entrant foams. Scarpa et al. [24] presented a novel 2D Voronoi-type cellular material with in-plane anisotropic behaviour; their elastic model can exhibit negative Poisson's ratio under uniaxial tensile loading. However, the cells of their model have shapes that are appropriate for certain types of honeycomb but are not readily extensible to represent cells of 3D foam materials with negative Poisson's ratio. Also, this 2D model was constructed by cells with only re-entrant shapes of bow-tie shape, and has anisotropic properties. By contrast, 3D foam materials with negative Poisson's ratio[9, 10], however, have cells with both concave (re-entrant) and convex shapes in a 2D view, and can have isotropic properties.

In this study, a modified Voronoi tessellation technique was introduced and was combined with FEM to build 2D random cellular solid models with re-entrant structures via a sequence of biaxial compressive plastic deformations. Then, the Poisson's ratios and energy absorption capacities of cellular solid models with different initial relative densities (0.032 and 0.039) were studied and discussed for different area compression ratios. The modeling work involved permanently deforming a Voronoi structure, and then computing elastic quantities for this new deformed structure. A minimum Poisson's ratio of approximately -0.38 was found at an appropriate permanent compression strain. A fitting rule which fits the 2D results to 3D space was established to fit the simulation to experimental results. The energy absorption capacities of cellular solids with different area compression ratios were also studied. The stress-strain diagrams of cellular solids showed that cellular solids with minimum negative Poisson's ratio exhibited the highest energy absorption capacity. Furthermore, mechanical properties of the random cellular solid model were compared with 2D regular honeycomb models with both concave and convex shaped cells. Results showed that the energy absorption capacity of the three models increased with increasing dynamic compression velocity. The random foam model exhibited the highest rate of increase in energy absorption capacity with increasing compression velocity.

Materials and methods

2D open cell cellular solids model with negative Poisson's ratio were generated using the technique of Voronoi tessellations to study the cellular solids mechanical behaviours. Firstly, a regular 2D Voronoi model was generated in Matlab according to the procedure described in Ref [17], and the irregularity of the Voronoi model was taken to be 0.45 for corresponding with the geometric dimensions of experimental foams, please see Fig. 1a. The irregularity describes the randomness of the cells in terms of shape and size. For a regular lattice, the irregularity equals 1; for a completely random Voronoi model, the irregularity equals zero (The Poisson's ratio has a minimal change with an increasing irregularity[19]). However all the cells of Voronoi model are convex polygons, different from the real foams which have both convex and concave cells. Therefore, we modified the regular 2D Voronoi models by merging any two adjacent polygons in Voronoi diagram into a new polygon by removing their common edge, and meanwhile we ensure no more than one edge was removed for the same polygon. The model was then imported in the commercial finite element software ANSYS/LS-DYNA by exporting the information of nodes and lines from the model in Matlab, as shown in Fig. 1b.

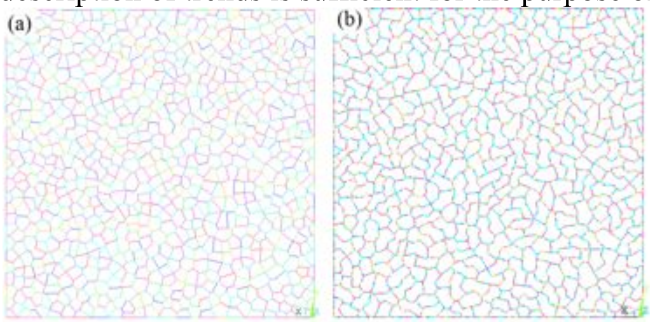
The size of the model was 35x35mm^2 with 374 cells. The properties of parent material were selected to be those of Cu alloy. The density, Young's modulus, Poisson's ratio were 8930 kg m^{-3}, 117GPa, 0.35, respectively. A bilinear strain-hardening relationship (stress strain diagram consists of two segments of different slopes to represent the elastic and plastic regions) was used to represent the true stress–strain relationship of the parent material and the yield strength and tangent modulus were 400MPa, 100MPa, respectively. The cell walls were meshed with Shell 163 elements(the number of elements depend on the edge length of the cell wall, every edge had at least 5 elements). The model was constrained in the z-direction and only in-plane motion of the nodes were permitted. The relative density values (defined as the ratio of area occupied by solid material to the total area) of the models, 0.032 and 0.039, were chosen by defining cell wall thickness (0.0384mm, 0.0468mm, respectively).

Two moving rigid platens were attached to the model, and the model was compressed from top and right with a constant velocity (2m/s) at the same time, as shown in Fig.2. Automatic single surface contact was applied to all the cell surfaces, and the surface to surface contact was defined between the rigid platens and Voronoi model. The information of elements and nodes, exported by using LS-PrePost at different area compression ratios A_r (A_r refers to the ratio of the area of the original model to that of the squeezed model), was imported in ANSYS to calculate the Poisson's ratio which was calculated in the elastic region of the stress strain diagrams from the FE analysis according to the definition of Poisson's ratio.

ANSYS/LS-DYNA was used to analyze the energy absorption capacity of the cellular solid models with different area compression ratios. The cellular solid models were compressed between two rigid platens; the bottom platen was fixed, and the top platen was set to move at a constant velocity. The dynamic friction coefficient between the edges of the model and the rigid platens was assumed to be 0.15[25].

Negative Poisson's ratio Voronoi models were prepared by biaxial permanent compression; for comparison, regular honeycombs with convex and concave cells were prepared by design. The density of these cellular solids later analyzed in compression was 348 kg/m^3.

It is worth to mention that using normal computers to run a random 3D model will lead to significant time expense, therefore, only 2D models were studied in the present work. However, the present 2D model results were qualitatively similar to 3D experiments, and a qualitative description of trends is sufficient for the purpose of the present study.



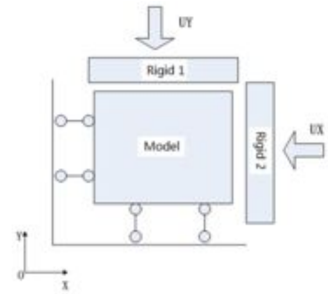


Fig.1 2D regular Voronoi and Modified Voronoi cellular solid structure

Fig.2 Biaxial quadrature squeezing constraint condition

Results and discussion

Poisson's ratio

The relationships between Poisson's ratio and area compression ratio of the cellular solids with two different initial relative densities of 0.032 and 0.039 are shown in Fig. 3. It can be seen from Fig. 3 that the Poisson's ratio of the cellular solids decreased at first to a minimum value and increased subsequently with an increasing area compression ratio. The minimum Poisson's ratios of both cellular solids were approximately -0.38, and the corresponding area compression ratio values were 5.459 and 4.695, respectively. Simulation results resemble those of experiments upon 3D foams, yet the simulated minimum Poisson's ratios were smaller than experimental observation and occurred at different area compression ratios from those determined from experiments. This phenomenon is attributed to the fact that simulation was performed on a 2D model, whereas, the experiments were conducted on 3D foams. As a result, we developed an empirical equation which can fit the 2D simulation results to 3D space as follows:

$$\begin{cases} V_r = 0.74A_r + 0.26 \\ \nu' = 1.35\nu - 0.17 \end{cases} \tag{1}$$

in which V_r is the volumetric compression ratio, A_r is the area compression ratio, ν' is the Poisson's ratio of 3D foams, ν refers to the Poisson's ratio of 2D foams, respectively.

The procedure of establishing the empirical equations and coefficients is as follows:

(1) Move all the points of the 2D results along the negative direction of x- and y-axis for a distance of $A_{r\min}$ and $\nu_{\min}$, respectively, where $A_{r\min}$ and $\nu_{\min}$ refer to the x- and y- values corresponding to the minimum Poisson's ratio of the 2D model;

(2) Reduce the x-values of the points of the 2D results after moving by a factor of n, and magnify the corresponding y-values by a factor of *n*. *n* = 1.35 was selected which best fits the experimental results;

(3) Move all the points a distance of Δx and Δy along the positive directions of the x-axis and y-axis, where Δx and Δy refer to the differences of x- and y- values between the first point in the 3D model (V_r=1, ν' =0.25). Motion was calculated using the formula:

$$\begin{cases} \Delta x = 1 - \dfrac{1 - A_{\mathrm{rmin}}}{n} \\ \Delta y = 0.25 - \dfrac{0.31 - \nu_{\min}}{n} \end{cases} \tag{2}$$

Then, the final equations and coefficient are established, and shown in Eq. 1.

The fitting curves of the empirical 3D form from 2D analysis to the experimental results for 3D foams are shown in Fig. 3(dash lines). It can be seen from Fig. 3 that the simulated 3D foam results match the experiment results very well in terms of both minimum Poisson's ratio values and the corresponding compression ratios.

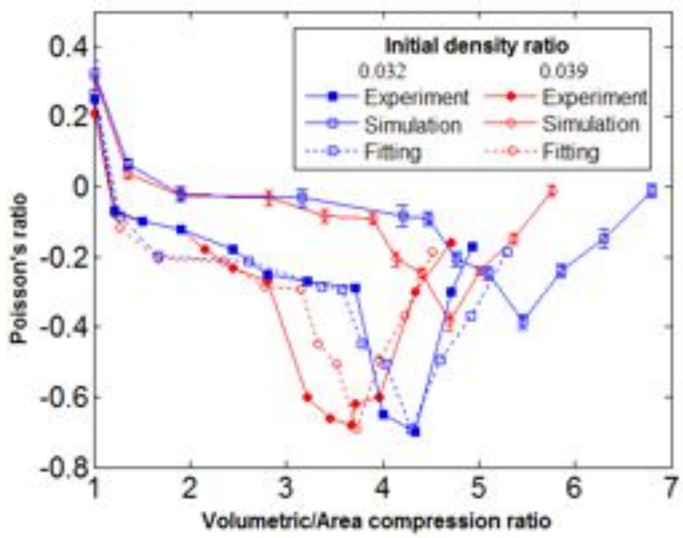


Fig.3 The relationship between Poisson's ratio and volumetric/area compression ratio

The microstructures of numerical models and the actual copper foams are compared as shown in Fig. 4. Fig. 4(a-c) show the microstructures of numerical models with different area compression ratios of 1, 4.478, 5.099. The actual copper foam microstructure images at different volumetric compression ratios of 1, 4.34 and 4.94 are shown in Fig. 4(d-f). It can be seen that excellent agreement was achieved between simulation and experimental observations, which demonstrates the feasibility of our numerical approach. Sufficient amount of permanent compression results in an inward bulging (re-entrant) cell structure. Extensive permanent compression will cause cell ribs to contact, so that the Poisson's ratio of the foam will increase.

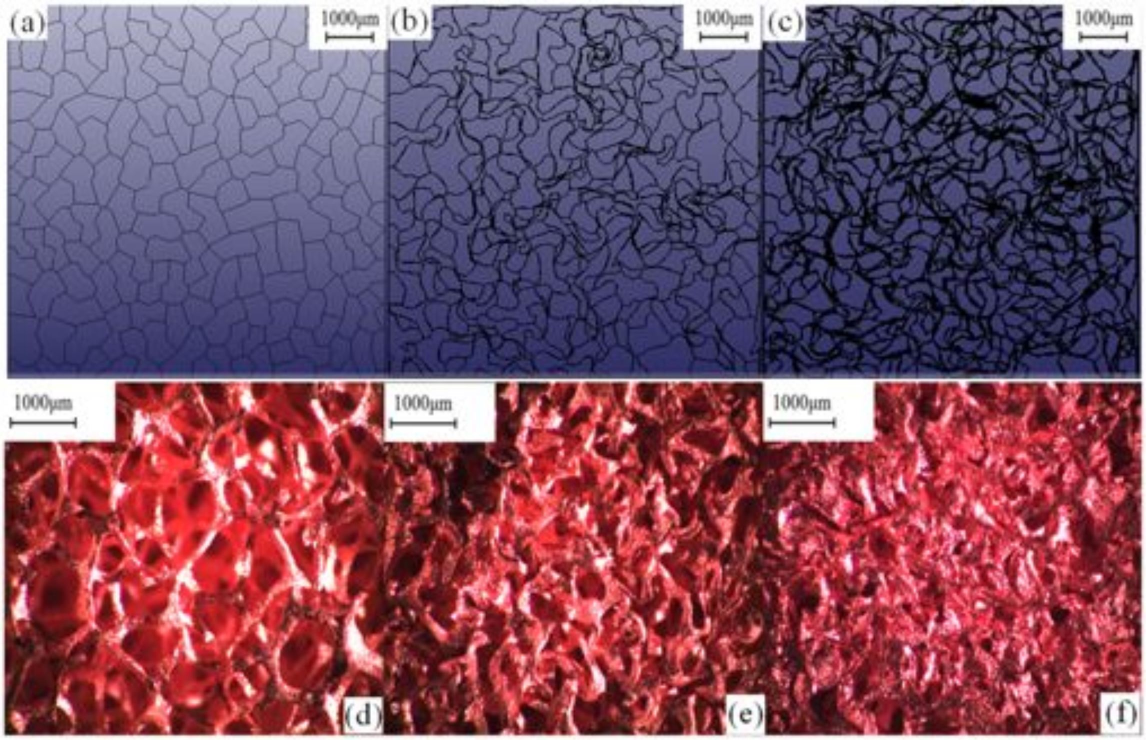


Fig.4 The comparison of microstructures of numerical model and the actual copper foams at different compression ratios. This copper foam has an initial relative density of 0.032. Fig. 4(a-c) are the microstructures of numerical models when A_r=1, 4.478, 5.099, and Fig. 4(d-f) show the microstructure images of a portion of the actual copper foam at different volumetric compression ratios of 1, 4.34 and 4.94 [10]

Energy absorption capacity

The energy absorption capacities of the cellular solid models with two different initial relative densities of 0.032 and 0.039 were analyzed at different area compression ratios, as shown in Fig. 5.

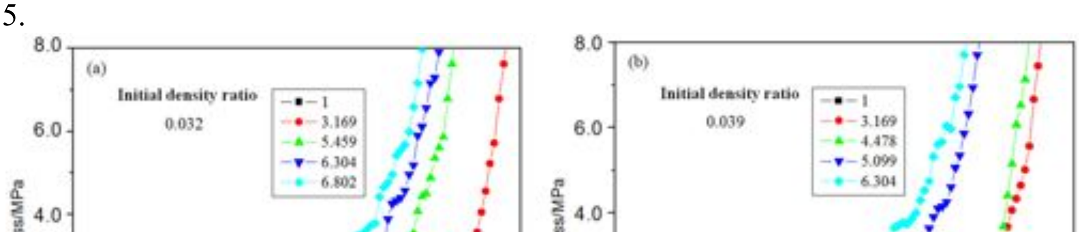


Fig. 5 The stress -strain curves of model material with two different initial relative densities (a) 0.032 and (b) 0.039 at different area compression ratios

The plateau stress and densification strain are the main parameters to characterize the nonlinear properties of cellular solids. The energy absorption per unit volume, W_v, is defined in Eq. (3) as

$$W_v = \int_0^{\varepsilon_D} \sigma(\varepsilon)\mathrm{d}\varepsilon \tag{3}$$

where $\sigma(\varepsilon)$ is the flow stress of the structure; ε_D is the densification strain[26]. The energy absorption per unit volume of the cellular solids shown in Figure 5 was calculated based on Eq.

(3). The results have been shown in Table 1. Results show that cellular solids with minimum negative Poisson's ratio can exhibit the highest energy absorption capacity.

Table 1 The Poisson's ratio and energy absorption per unit volume of the cellular solid models with different area compression ratios at two different initial relative densities

	Initial density ratio									
	0.032					0.039				
A_r	1	3.169	5.459	6.304	6.802	1	2.819	4.695	5.020	5.757
v	0.31	-0.03	-0.38	-0.17	-0.01	0.31	-0.02	-0.38	-0.24	-0.01
W_v/MJm^{-3}	0.064	0.197	0.291	0.280	0.288	0.089	0.228	0.399	0.302	0.357

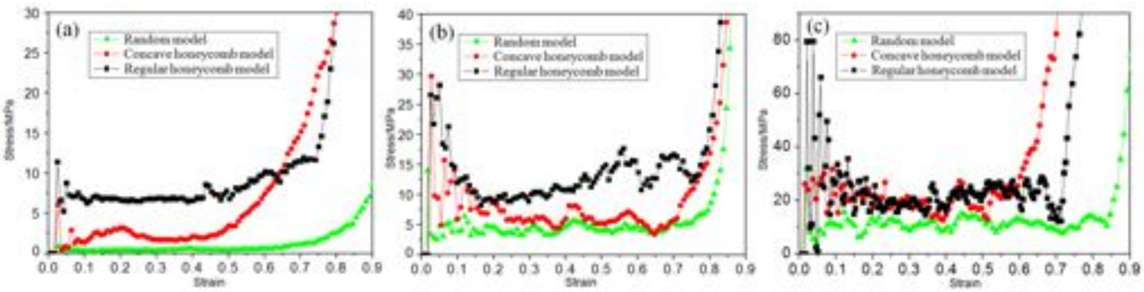


Fig. 6 The stress-strain curves of the random foam model, 2D regular honeycomb model and concave honeycomb model at different ompression velocities: (a) v=10m s^{-1}; (b) v=60m s^{-1}; (c) v=100m s^{-1}

It is interesing to compare the mechanical properties of the random 2D Voronoi model with regular honeycomb structures. 2D regular honeycomb models with both concave and convex shaped cells with the same initial density ratio (0.032) as the random model at A_r=5.459 (with the minimum negative Poisson's ratio) were built, and the energy absorption capacities of these models were analyzed at different dynamic compression velocities; their stress-strain diagrams are shown in Fig. 6. The deformation mechanisms of these models at a compression velocity v =60m s^{-1} are shown in Fig. 7.

We can see from Fig. 7 that the "X"-shaped and the "V"-shaped localized bands were formed at the crushing edge of the regular honeycomb model and the support edge of the honeycomb model with concave shaped cells, respectively. The random model collapsed step by step from the crushing edge, and no regular localized bands pattern was observed during the compression deformation. Bands of localized deformation occur in honeycomb and foam that is compressed in one direction[27].The observation of band formation during compression has been analyzed as a manifestation of instability [27]during the buckling of the ribs in the cellular solids. The elastic instability observed in the foams was associated with non-monotonic load deformation characteristic of the individual cells, hence negative stiffness. It is notable in Figure 4 a-c that biaxial compression of the Voronoi model does not give rise to any bands of heterogeneous deformation. By contrast in Figure 7b, uniaxial deformation of a model of regular hexagons does cause such bands. The Voronoi model in Figure 7a and the negative Poisson's ratio honeycomb in Figure 7c exhibit densification near the top constraint surface. This asymmetry of deformation is attributed to the dynamic character of the imposed motion. Buckling of cells under uniaxial deformation has a different character in comparison with multiaxial deformation[27]. Such a distinction is considered responsible for the absence of bands in the biaxial compression in Figure 4 a-c; also for non-monotonic deformation characteristic observed in 3D foams [28].

Values of the energy absorption per unit volume of the random foam model, 2D regular honeycomb models with concave and convex shaped cells at different compression velocities have been shown in Table 2. Results showed that the energy absorption capacity of the three models increased with an increasing dynamic compression velocity, a phenomenon consistent with those reported by other researchers in negative Poisson's ratio foams [29, 30]; however, the random foam model exhibited the maximum rate dependence of the dynamic energy absorption capacity on the dynamic compression velocity. At v =100m s^{-1}, the energy absorption capacity of the random foam model was better than that of the honeycomb model with concave shaped cells due to the long plateau region before densification.

Table 2 The energy absorption per unit volume of the random foam model, 2D regular honeycomb models with both concave and convex shaped cells under different compression velocities.

Compression velocity(m/s)	W_v/MJm^{-3}		
	Random foam model	Honeycomb model with concave cells	Regular honeycomb model
10	0.399	1.360	5.069
60	3.608	5.285	10.221
100	10.005	10.004	15.716

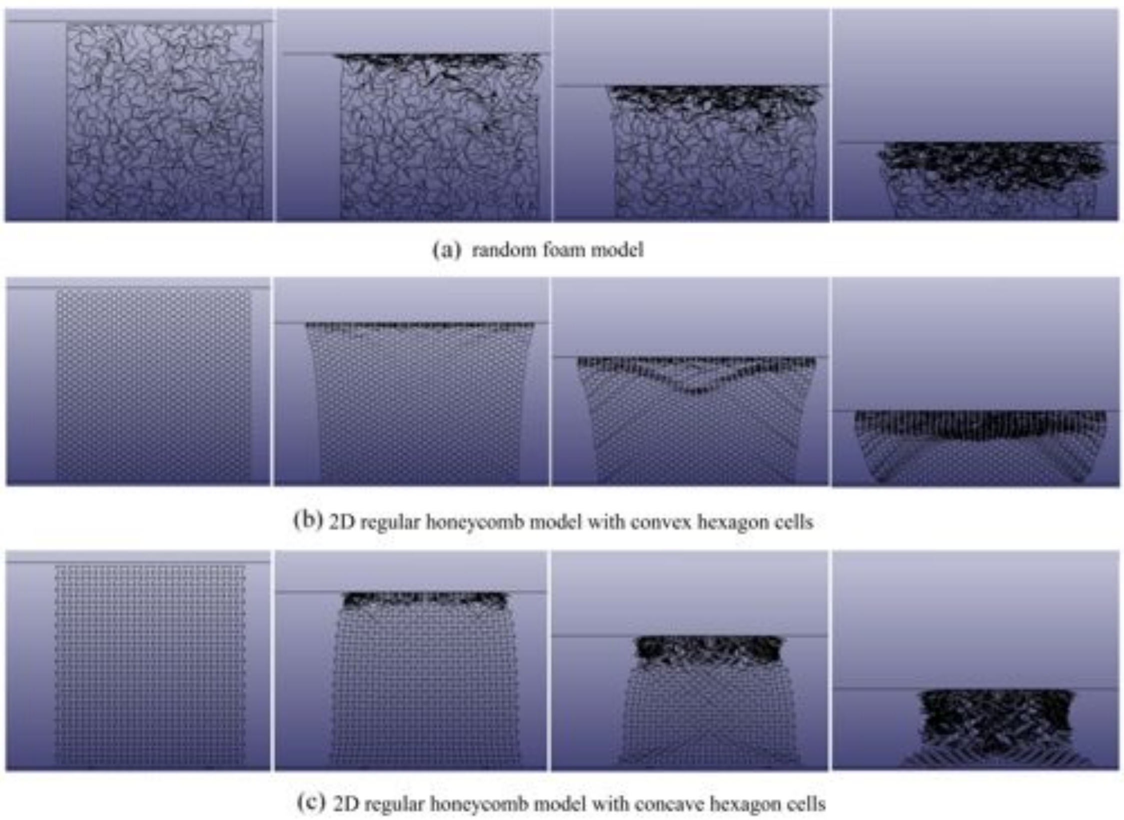


Fig. 7 Deformation mechanisms at a compression velocity v=60m s^{-1} (a) random foam model (A_r=5.459); (b) 2D regular honeycomb model with convex hexagon cells; (c) 2D regular honeycomb model with concave hexagon cells.

Fig. 7 shows the deformation mechanisms of random foam model (A_r =5.459) and 2D regular honeycomb models with convex and concave shaped cells at a compression velocity v =60m s^{-1}.

Conclusions

A series of 2D random cellular solid models with re-entrant structures were built using FEM(finite element method) based on a modified Voronoi tessellation technique. The Poisson's ratio and energy absorption capacity of cellular solid models with different initial relative densities (0.032 and 0.039) were studied at different area compression ratios. A minimum in Poisson’s ratio of approximately -0.38 was achieved with appropriate area compression ratios. An empirical rule was established which best fits the 2D simulation results to the 3D experimental results. The cellular solids with the minimum negative Poisson's ratio exhibited the highest energy absorption capacity. Furthermore, mechanical properties of the random cellular solid model were compared with 2D regular honeycomb models with both concave and convex shaped cells. Results showed that the energy absorption capacity of the three models increased with an increasing dynamic compression velocity; yet, the random foam model exhibited the highest increase rate in energy absorption capacity with the increasing compression velocity.

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Compliance with ethical standards
Conflict of interest The authors declare that they have no conflict of interest.

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