The time-dependent Poisson's ratio of viscoelastic materials can increase or decrease

by

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1 Summary

In viscoelastic materials, the Poisson's ratio is not a material constant but can depend upon time. For polymeric solids, the shear modulus relaxes much more than the bulk modulus, therefore, the Poisson's ratio ν(t) is an increasing function of time. In this article we demonstrate that such time dependence is not a necessary consequence of the theory of viscoelasticity. Composite microstructures are presented which result in ν(t) which decreases with time.

2 Poisson's ratio in viscoelastic materials

The time dependence of Poisson's ratio in linearly viscoelastic materials can be examined via the correspondence principle. For an elastic material, the Poisson's ratio ν is given by

$$\nu = \frac{1}{2} - \frac{E}{6B} = \frac{1}{2} - \frac{E\kappa}{6} \qquad (1)$$

in which E is Young's modulus, B is the bulk modulus and $\kappa = 1/B$ is the compressibility. Applying the correspondence principle,

$$p\nu(p) = \frac{1}{2} - \frac{pE(p)}{6pB(p)} = \frac{1}{2} - \frac{p^2E(p)\kappa(p)}{6} \qquad (2)$$

in which p is the Laplace transform variable and functions of p are Laplace transforms of corresponding functions of time t.

Consider a polymeric material in which the shear relaxation modulus G(t) may vary by more than a factor of 1,000 over the full range of time t, and the bulk relaxation modulus B(t) varies much less, perhaps a factor of two. Approximate B(t) as a constant, and transform back into the time domain. Then,

$$\nu(t) = \frac{1}{2} - \frac{E(t)}{6B} \qquad (3)$$

Since E(t) is a monotonically decreasing function, ν(t) is an increasing function of time for these polymeric materials.

It is natural to ask whether this is true in general. The general case may be examined in an explicit form by considering the compressibility $\kappa = 1/B$ in Eqs. 1 and 2. Again transforming back to the time domain,

$$\nu(t) = \frac{1}{2} - \int_0^t E(t-\tau)\,\frac{d\kappa(\tau)}{d\tau}\,d\tau \qquad (4)$$

Young's relaxation modulus E(t) is a decreasing function and the compressibility κ(t), as a creep compliance, is an increasing function. The rates of relaxation and creep depend on the particular material, so no conclusions regarding ν(t) are apparent from the general relation Eq. (4).

We now consider composite microstructures which give rise to a Poisson's ratio which decreases with time. To simplify the arguments, we consider only limiting values of material properties at very short and very long times. The honeycomb structure shown in Fig. 1 contains elastic and viscoelastic ligaments represented by solid and dashed lines respectively. At short times, both kinds of ligaments are assumed to have the same stiffness. The resulting structure is a triangulated one with a positive Poisson's ratio. At long times the viscoelastic ligaments have relaxed to zero stiffness. The structure has effectively become a re-entrant honeycomb with a negative Poisson's ratio; if the triangles in Fig. 1 are equilateral, $\nu = -1$. Consequently, the Poisson's ratio of the structure decreases with time in contrast to the polymer material considered above. Composite materials based on other types of cellular negative Poisson's ratio structures (Lakes, 1991) are also possible. A hexagonal structure of that type is manifestly isotropic in its elastic properties, therefore anisotropy is not required to achieve the above effects.

As for three dimensional viscoelastic composites, we envisage a negative Poisson's ratio elastic foam skeleton (Lakes, 1987) in which microcellular viscoelastic foam with a conventional cell structure has been injected into the interstices. For short times, the viscoelastic component is assumed to be stiff enough so that it provides most of the stiffness of the composite. Consequently the Poisson's ratio approximates that of a conventional foam: about 1/3 (Gibson and Ashby, 1987). For long times, we assume that the stiffness of the viscoelastic component tends to zero, so the mechanics of the composite are dominated by the re-entrant foam skeleton, which can have a Poisson's ratio as small as 0.7. Therefore, the Poisson's ratio decreases with time as in the two dimensional case.

The viscoelastic Poisson's ratio $\nu(t)$ need not be monotonic in time. A composite with non-monotonic behavior may be constructed by using in the above examples a viscoelastic conventional constituent which relaxes to a small but finite stiffness. The negative Poisson's ratio skeletal constituent is also made viscoelastic, with a relaxation time much longer than that of the conventional constituent. The Poisson's ratio of such a composite first decreases with time, then increases.

3 Implications of a time varying Poisson's ratio

Poisson's ratio governs various three-dimensional stress fields including those associated with stress concentrations, as well as deformation kinematics such as the transverse curvature associated with bending. Consequently, a time varying Poisson's ratio in a viscoelastic composite will result in changing stresses and deformations. For example an anticlastic (saddle shaped) curvature of a viscoelastic composite in response to bending could with time change to a synclastic (ellipsoidal) curvature; or the reverse could occur, depending on the material. One may also envisage fasteners held by interference fit in which the fastening force increases with time or disappears entirely.

4 Conclusions

The Poisson's ratio of viscoelastic material can increase or decrease with time. Composite structures have been presented which exhibit a decrease of Poisson's ratio with time, in contrast to the situation in homogeneous polymers.

References

Gibson, L. J. and Ashby, M. F., "Cellular Solids" (Pergamon Press, 1988)

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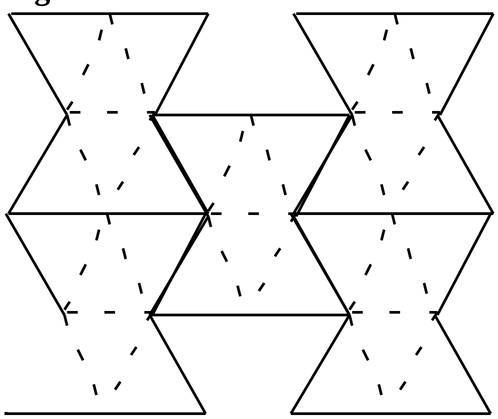
Figures



1 Viscoelastic honeycomb structure. It exhibits a decreasing Poisson's ratio provided that the solid lines represent elastic ligaments and dashed lines represent viscoelastic ligaments which relax with time to zero stiffness.