The resonant ultrasound spectroscopy method for determining the Poisson's ratio of spheres over the full range

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Abstract:

In this paper, the method for determining the Poisson's ratio of isotropic spheres is studied via resonant ultrasound spectroscopy (RUS). To that end, The mode structure maps for freely vibrating isotropic spheres are obtained via finite element method over the full range of Poisson's ratio (-1 to +0.5). RUS experimental measurements for spherical materials (Pure indium, steel, SiO_2, 13.5wt% In-Sn alloy, and copper foams) are compared with the numerical results and the Poisson's ratio is determined as +0.4, +0.3, +0.2, -0.08, and -0.3, respectively. The effects of slight imperfection of specimen shape on the first 12 modes are analyzed for various Poisson's ratios.

Keywords: Negative Poisson's ratio; alloys; spectroscopy; porous materials; sphere

1. INTRODUCTION

Poisson's ratio (ν) is defined as the ratio of the transverse contraction strain to the longitudinal extension strain in tension. Poisson's ratio can range from -1 to 0.5 for isotropic materials, but most solids have positive Poisson's ratio (0.25-0.33). Negative Poisson's ratio is known in single crystals and also occurs in designed foams via unfolding of the cells [1]. Negative Poisson's ratio behavior has been observed experimentally in polymer gels near the phase transitions [2, 3], orthorhombic alloy in a set of planes [4], ferroelastic ceramic [5] and InSn alloy [6] near the phase transformations.

RUS [7] is known as a useful technique to determine elastic or viscoelastic moduli by measuring the resonance structure of specimens with compact shapes such as cubes, spheres, and short cylinders [8]. Yaoita [9] provided a diagram of modal frequency as a function of Poisson's ratio for isotropic spheres of Poisson's ratio between 0 and 0.45. With such a plot, one can easily extract Poisson's ratio as well as modulus and damping without numerical inversion[10]. However, with the development of negative Poisson's ratio materials, it is necessary to obtain a plot over the full range of isotropic Poisson's ratio.

In this work, the mode structures for a isotropic sphere is determined numerically for the full range of Poisson's ratio from -0.98 to 0.48. The Poisson's ratios of steel, SiO_2, re-entrant copper foam, pure Indium and 13.5wt% In-Sn alloy spheres were determined experimentally via RUS. Experimental measurements were compared with the numerical results, and good agreement was found between the measurements and predictions. The effect of slight ovality was also evaluated numerically for materials with various Poisson's ratios.

2. NUMERICAL ANALYSIS

The mode structures as a function of isotropic Poisson's ratio of an isotropic solid spheres were studied using the commercial finite element software ANSYS with the 3D deformable sphere models. Poisson's ratio was set to vary from -0.98 to 0.48 with an incremental step of 0.1. Mapped meshes of 32000 hexahedral elements (Solid 185, 8 nodes) were used. Mode shapes and frequencies were determined using modal type of analysis; Block Lanczos was selected as the mode extraction method for the first 200 modes.

3. EXPERIMENTAL

Spherical samples were prepared using five different materials, steel, fused silica (amorphous SiO_2; Technical Glass, Painesville, OH), 13.5wt%In-Sn alloy (a composition near the phase boundary; alloy fabrication method can be found in [6]), pure indium (99.9%, TED PELLA, Inc., Redding, CA), and open cell copper foams (Astro Met Associates, Inc., Cincinnati, OH), with a diameter of 30.0mm, 29.2mm, 9.1mm, 23mm, 10.0mm, respectively. The as-received fused silica was in spherical shape; metal samples were cut into sphere with a CNC mill. The copper foams were initially squeezed from the orthogonal directions to get re-entrant copper foams with negative Poisson's ratio [11].

The RUS samples were supported by two transducers (as shown in Fig. 1) with minimal force to approximate the free vibration condition assumed in the analysis. Transducers used were Panametrics V153 1.0/0.5 broadband shear, polarized with center frequency 1 MHz and longitudinal (compressional) 1 MHz transducers. The driver transducer was excited via a synthesized function generator, and the output was amplified by a preamplifier, signals were captured by a digital oscilloscope. The band-pass used was from 100 Hz to 300 kHz.

4. RESULTS AND DISCUSSION

4.1. Numerical results

Fig. 1 shows the plot for an isotropic sphere over the full range of isotropic Poisson's ratio from -1 to 0.5. Frequencies are normalized by f_0 [9]:

$$f_0 = \frac{1}{2\pi R}\sqrt{\frac{G}{\rho}} \tag{1}$$

where R, G and ρ are the radius, shear modulus and density of the sphere. The fundamental torsional mode appears in the graph at a normalized frequency $f/f_0 = 2.5$.

Fig. 1 shows the first 12 modes. Discrete values of Poisson's ratio chosen give rise to kinks in the curves. The results agree well with Yaoita's plot in the corresponding range of Poisson's ratio.

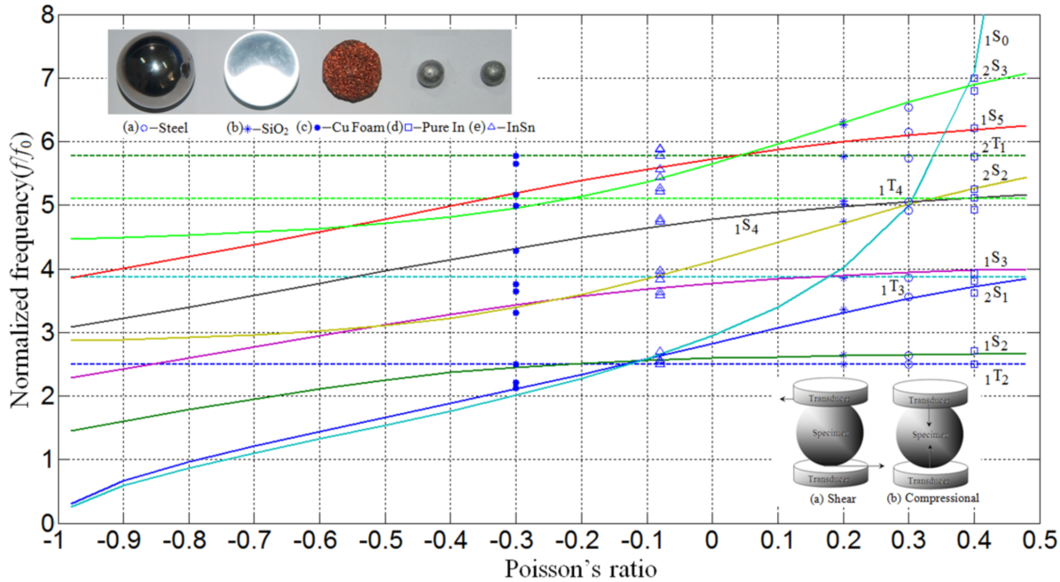


Fig. 1 shows that the fundamental mode for an isotropic sphere is the torsional mode (i.e., $_1T_2$) when the Poisson's ratio is between -0.13 and +0.48. For spheres with a Poisson ratio below -0.13, the fundamental mode is a predominantly spherical mode (i.e., $_1S_0$). Modes $_1S_0$, $_2S_2$ and $_2S_1$ decrease rapidly in frequency with decreasing Poisson's ratio. The mode $_2S_1$ always shows up as one of the first three modes, and its rapid change as a function of Poisson ratio compared with the other two first modes (i.e., ${}_1T_2$ and ${}_1S_2$) allows for its easy identification. ${}_1S_2$ is the second mode when -0.98 < ν< -0.13, and the third mode when -0.13 <ν< 0.48. The torsional modes (broken line) are insensitive to Poisson's ratio's since the torsional modes are determined by shear deformation only. However, the spheroidal modes (solid line) are determined by both shear and compression deformation, therefore they are sensitive to Poisson's ratio.

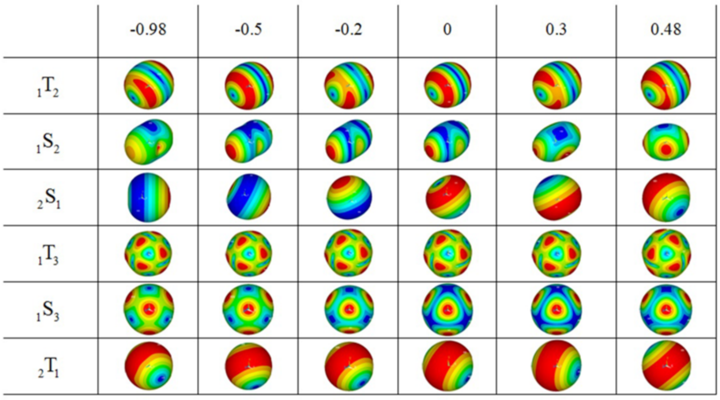


Fig. 2 shows the representative mode shapes of isotropic spheres and their dependence on Poisson's ratio for modes ${}_1T_2$, ${}_1S_2$, ${}_2S_1$, ${}_1T_3$, ${}_1S_3$ and ${}_2T_1$. Colors represent the magnitude of displacement with dark blue refers to minimum displacement, and red the maximum displacement. the intermediate displacements appear as yellow and green.

In reality, slight deviation in shape from an ideal sphere inevitably exists. Therefore, it is helpful for the interpretation of experimental results by investigating the effect of ovality on the mode structures.

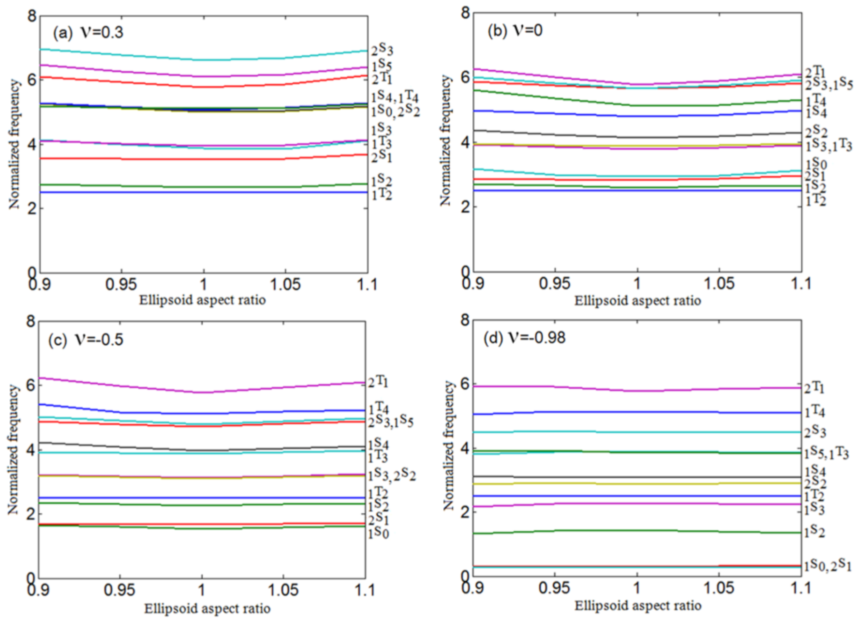
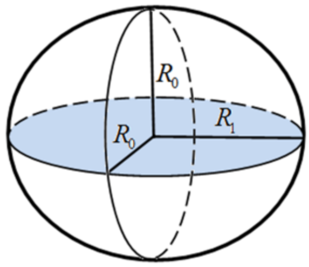


Fig. 3 shows the normalized frequencies of the first 12 modes of an isotropic spheroid with slight ovality as a function of the aspect ratio R_1/R_0 for materials with various Poisson's ratios. The aspect ratio R_1/R_0 is defined in Fig. 4.



As shown in Fig. 3, most modes tend to shift to lower frequencies when the aspect ratio R_1/R_0 deviates from 1, but the decreasing rate is different from one mode to another. Modes ${}_1T_2$ and ${}_2S_1$ are insensitive to R_1/R_0 ratio. Close examination of Fig. 3 suggested that slight ovality has a very small effect on the experimental results since the relative positions of the modes do not change.

4.2. Experimental results

The mode structures of steel, SiO_2, open cell Cu foam, InSn alloy and pure indium sphere samples were plotted in Fig. 1 and compared with the numerical results. Frequencies were normalized to the first torsional mode frequency. The first torsional modes were found to be the lowest mode of steel and silica spheres with a frequency of 85.4kHz and 92.6kHz, respectively, and the corresponding shear modulus derived from Equation (1) is 80.8GPa and 29.0GPa, respectively; the shear moduli are consistent with published results [12]. Compared with the numerical results, the Poisson's ratios of steel and SiO_2 were determined to be +0.3 and +0.2 respectively. For the re-entrant copper foam specimen with volumetric compression ratio of 2.4, the first torsional frequency and the calculated shear modulus is 18.04kHz and 447.6MPa, respectively. A non-torsional mode was observed below the first torsional mode, confirmed by the observation that this mode showed up by using both compressional and shear transducers. The Poisson's ratio of the re-entrant copper foam specimens was determined to be about -0.3; this is consistent with the recently reported value [12]. The shear modulus and Poisson's ratio of the InSn alloy were determined to be 17.6GPa and -0.08 respectively. The results are also consistent with recently reported values [6]. This alloy is highly sensitive to small changes in composition, so some variation is expected. Confirmation for the negative Poisson's ratio materials is as follows. For the copper foams, similar foams were studied using optical methods [11] with similar results. For the InSn alloys, RUS measurements were also done [6] using a cylindrical shape, with identification of modes.

5. CONCLUSIONS

Experimental results are in good agreement with the numerical results in terms of the mode structures of spheres with Poisson's ratios of +0.3, +0.2, -0.08 and -0.3. Slight splitting of some modes (less than 5%) was observed and considered to be attributed to the material anisotropy. However, modes can be identified by the difference in response to the longitudinal and shear excitations; therefore, mode splitting did not interfere with interpretation. The effects of slight sample ovality on the first 12 modes are also studied for various Poisson's ratios, and are found to be negligible in interpretation of the experimental results.

Acknowledgements

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Figure caption

Fig. 1 Normalized frequencies of an isotropic sphere over the full range of isotropic Poisson's ratio from -1 to 0.5. The capital letters ${}_iT_n$, and ${}_iS_n$ refer to the torsional and spheroidal mode respectively; the subscripts "*i*" and "*n*" refer to the number of modes in the radial direction of sphere and the order of spherical Bessel functions, respectively. Modal frequencies are normalized to the fundamental frequency. Experimental data are indicated as (○), (*), (□), (□)and (●) which represent steel, SiO_2, InSn alloy(13.5%In, air cooled), pure indium and Cu foam (Cu foam squeezed by a volumetric compression ratio of 2.4), respectively.

Fig. 2 (Color online) Representative mode shapes and their dependence on Poisson's ratio for the modes ${}_1T_2$, ${}_1S_2$, ${}_2S_1$, ${}_1T_3$, ${}_1S_3$ and ${}_2T_1$. Colors represent magnitude of displacement; for interpretation of color (or shading), refer to the text.

Fig. 3 Normalized frequencies for an isotropic ellipsoid as a function of aspect ratio R_1/R_0 for materials with various Poisson's ratios. (a) ν=0.3;(b) ν =0,(c) ν =-0.5;(d)ν =-0.98.

Fig. 4 The ellipsoid model