Shape-Dependent Damping in Piezoelectric Solids

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Abstract—The piezoelectric contribution to the mechanical loss tangent of a piezoelectric solid is derived from its complex piezoelectric and dielectric coefficients. This loss depends on specimen geometry as a result of differences in effects related to the electrical boundary conditions. Inclusion of a positive out-of-phase piezoelectric modulus results in reduced values of the predicted loss, which constitutes an improvement over earlier theories which predict losses exceeding measured losses by a factor greater than two.

I. Introduction

THE PRESENCE of relaxation effects in the dielectric and mechanical behavior of real materials is well known; such effects can be described by complex material coefficients. Piezoelectric relaxation has been observed in a variety of materials, including ceramics [1], [2], composites [3], and bone [4]. Such relaxation can be represented by means of complex piezoelectric coefficients or by a piezoelectric loss tangent [1], [2], [5]. Piezoelectric phase angles are of some importance in the accurate determination of the piezoelectric coefficients of solids [6]. Mechanical relaxation also occurs in piezoelectric materials and is an important consideration in the application of such materials: large damping is considered desirable in materials used to generate short acoustic pulses for flaw detection [7]; small damping (high mechanical Q) is desirable in stable resonators and high power transducers.

In this paper the connection between the dielectric, mechanical, and piezoelectric coefficients of a material which exhibits relaxation is considered. Clearly, for an ideal solid which does not relax, this connection, in the form of a piezoelectric contribution to the compliance, has been established. For such a material, in the linear domain, the constitutive equations are [8]

$$D_i = [d_{ijk}]_T \, \sigma_{jk} + [K_{ij}]_{\sigma, T} \, E_j + [p_i]_\sigma \, \Delta T \tag{1}$$

$$\epsilon_{ij} = [S_{ijkl}]_{E, T} \, \sigma_{kl} + [d_{kij}]_T \, E_k + [\alpha_{ij}]_E \, \Delta T. \tag{2}$$

Here D is the electric displacement, d is the piezoelectric modulus at constant temperature T, K is the dielectric tensor at constant stress σ and temperature T, E is the electric field, p is the pyroelectric coefficient at constant stress, ϵ is the strain, S is the elastic compliance at constant field, and α is the thermal expansion. The usual Einstein summation convention over repeated suffices is used. In a material described by these equations, the isothermal compliance measured at constant electric displacement $[S_{ijkl}]_D$ differs from the compliance measured at constant field $[S_{ijkl}]_E$ [8]:

$$[S_{ijkl}]_D - [S_{ijkl}]_E = -d_{mij} d_{nkl} \, [K_{mn}]_\sigma^{-1}. \tag{3}$$

This difference may be regarded as the piezoelectric contribution to the compliance. Piezoelectric reactions also influence the apparent stiffness of a solid, under conditions in which neither E nor D is constant [9]. For materials which do relax, i.e., for which the coefficients in (1) and (2) are allowed to be complex,

$$d^*_{ijk} = d'_{ijk} - i d''_{ijk}$$
$$K^*_{ij} = K'_{ij} - i K''_{ij}$$
$$S^*_{ijkl} = S'_{ijkl} - i S''_{ijkl}, \tag{4}$$

the relationship between the coefficients has not been clearly defined. One can consider a piezoelectric contribution to mechanical relaxation: experimental evidence for such a contribution in quartz under quasistatic loading has appeared at least as early as 1915 [10]. A connection between dielectric loss and mechanical loss in piezoelectric solids is to be expected on heuristic grounds in that dielectric relaxation entails dissipation of electrical energy; if this energy has come from the piezoelectric conversion of mechanical energy, then mechanical relaxation or anelasticity must also occur. Several authors have developed theoretical expressions for the mechanical loss tangent $\tan \delta^{(s)} = S''/S'$ in terms of the dielectric loss tangent $\tan \delta^{(k)} = K''/K'$ and the piezoelectric coefficients. A summary of these expressions is shown in Table I. The results differ as a result of the use of different piezoelectric coefficients and as a result of different assumptions made. The latter difference is the more significant and will be considered in Section IV.

II. The Field-Displacement Relation

A. General Considerations

The relation between electric field and electric displacement is given by the constitutive equation (1). For a nonpolar solid under isothermal conditions, with $K_{ij} = k_{ij} e_0$ (e_0 is the permittivity of free space), and with $k^* = k' - ik''$ and $d^* = d' - id''$ to describe dielectric and piezoelectric relaxation, (1) and (2) become

$$D_i = d^*_{ijk} \, \sigma_{jk} + [k^*_{ij}]_\sigma \, e_0 E_j. \tag{5}$$

$$\epsilon_{ij} = [S^*_{ijkl}]_E \, \sigma_{kl} + d^*_{kij} E_k. \tag{6}$$

Suppose that the specimen in question is electrically isolated, i.e., free of any attached circuit element of finite impedance and that it is subjected to a stress $\sigma_{11} \neq 0$ which is uniform

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TABLE I

Investigator, Reference	Assumptions	Result: Piezoelectric contribution to anelastic loss
Hutson, White (1962) [11]	Ultrasonic wave. Single relaxation time. d" neglected.	$\tan\delta = \frac{e^2}{CK}\frac{\omega_c/\omega}{1+(\omega_c/\omega)^2}$; $e = d{:}C$; $\omega_c = \sigma/K$
Holland (1967) [5]	Phases arbitrarily set to zero. With d".	$\tan\delta_{1111} = \frac{1}{S'_{1111}}\frac{\left(d''_{311}\right)^2}{k'_{33}}$
Ogawa (1969) [12]	Waves and vibrations. d" neglected. L treated as an adjustable parameter except at its upper and lower bounds. Single dielectric relaxation time.	$\tan\delta = \frac{\left(d_{31}^2/S_{11}K'_3\right)L^2K'_3(\sigma/\omega)}{\left(1+L(K'_3-e_0)\right)^2+(L\sigma/\omega)^2}$
Yamaguchi, Takahashi [13]	Resonant vibrations. d" neglected.	$\tan\delta = \frac{\pi K_{33}g_{31}^2}{S'}\tan\delta^k$, $g = d{:}K^{-1}$
Present Study	Subresonant dynamic loading. Geometry and d" included. Thin disk, E ⊥ Surface: Λ = 0. Cylinder, E ⊥ Long Axis: Λ = 1. Sphere: Λ = 2.	$\tan\delta_{1111} = \frac{\tan\delta^k_{33}\left(d'^2_{311}-d''^2_{311}\right)-2\left(1+\Lambda/k'_{33}\right)d'_{311}d''_{311}}{\left(\left(1+\Lambda/k'_{33}\right)^2+\tan^2\delta^k_{33}\right)S'_{1111}e_0k'_{33}}$

Nomenclature

C: Elastic Modulus Tensor
d: Piezoelectric d Tensor
e: Piezoelectric e Tensor
g: Piezoelectric g Tensor
k: Dielectric Tensor
L: Depolarization Factor
σ: Conductivity
e_0: Permittivity of Free Space
S: Compliance Tensor
δ: Anelastic Loss Angle
δ^k: Dielectric Loss Angle
τ: Relaxation Time
ω: Angular Frequency

within the specimen and varies sinusoidally with time: $\sigma_{11}(t) = \sigma_{11}^{(0)} e^{i\omega t}$, $\sigma_{ij} = 0$ if $i \neq 1$ or $j \neq 1$. The supposition of uniform stress entails loading below any mechanical resonance. With these assumptions we shall find that the field and the displacement are related in a way which depends on the electrical boundary conditions; therefore, several specimen geometries will be considered separately. In all cases we assume that $d_{311} \neq 0$ and $d_{i11} = 0$ if $i \neq 3$ and that the 3 direction is a principal axis of k_{ij}, so that both the field and displacement will be in the 3 direction.

B. The Thin Plate

Consider a thin plate of piezoelectric material such that the flat surfaces are perpendicular to the 3 axis. From Gauss's law, the boundary condition on the electric displacement is $D_{\text{in}}^{[\text{normal}]} - D_{\text{out}}^{[\text{normal}]} = \Sigma_{\text{free}}$; Σ_{free} is the density of free charge on the surface. For the thin plate geometry, $D_{\text{out}}^{[\text{normal}]} = 0$. Now the charge which accumulates on the surface as a result of conductivity or modes of dielectric relaxation which do not involve dipole rotation, may be regarded as free charge [14]. Such a view, while conceptually reasonable, leads to difficulties in gedanken experiments involving measurement of the electric field in a plate with constant d' and finite conductivity. As a result of such experiments, we conclude that in the present setting, free charge must include only charge that is not associated with processes included in the definition of D. So if k'' contains contributions from dc conductivity as well as dielectric relaxation, the free charge is zero. Then inside the plate,

$$D_3 = 0, \tag{7}$$

as in the case of ideal crystals with zero conductivity [9].

C. The Long Cylinder

The 1 axis in this case is the cylinder long axis; the 3 axis is orthogonal to the cylinder long axis. The electric displacement will be in the 3 direction and will be uniform, so div $D = 0$ within the solid. The volume charge density vanishes, so Laplace's equation may be used. In polar coordinates in the 3-2 plane, the solutions are based on cylindrical harmonics and have the form

$$E_{\text{out}}^{\theta} = A(a^2/r^2)\sin\theta \tag{8a}$$

$$E_{\text{out}}^{r} = A(a^2/r^2)\cos\theta \tag{8b}$$

$$E_{\text{in}}^{\theta} = E_{\text{in}}\sin\theta \tag{8c}$$

$$E_{\text{in}}^{r} = -E_{\text{in}}\cos\theta \tag{8d}$$

$$D_{\text{out}}^{\theta} = A(e_0 a^2/r^2)\sin\theta \tag{8e}$$

$$D_{\text{out}}^{r} = A(e_0 a^2/r^2)\cos\theta \tag{8f}$$

$$D_{\text{in}}^{\theta} = C\sin\theta \tag{8g}$$

$$D_{\text{in}}^{r} = -C\cos\theta, \tag{8h}$$

in which a is the radius of the cylinder. With the boundary condition on the normal component of electric displacement and (8f) and (8h),

$$-C\cos\theta - Ae_0\cos\theta = -\Sigma_{\text{free}} \tag{9}$$

the boundary condition on the tangential component of the electric field in the quasielectrostatic approximation is $E_{\text{in}}^{(t)} = E_{\text{out}}^{(t)}$; with this and (8a) and (8c), one obtains $A = E_{\text{in}}$. With (7),

$$-dC/dt - e_0 dE_{\text{in}}/dt = 0. \tag{10}$$

Since the field and displacement have a sinusoidal time dependence,

$$D_3 = -E_3 e_0, \tag{11}$$

within the cylinder of piezoelectric material.

D. The Sphere

Although the application of a uniform uniaxial stress to the interior of a solid sphere would be a challenge to the experimentalist, we consider this situation in order to examine the effects of geometry on depolarization. The solution for the sphere, consisting of appropriate zonal harmonics which satisfy the boundary conditions, parallels that for the cylinder; for example $E^{\theta}_{\text{out}} = A\,(a^3/r^3)\sin\theta$. The field inside the sphere is uniform. Within the piezoelectric sphere, we obtain

$$D_3 = -E_3 e_0 \Lambda. \tag{12}$$

The field-displacement relations have been obtained from the electrical boundary conditions at the surface of the solid and from the supposition that the field vanishes infinitely far from the solid, which is a consequence of the assumption of an electrically isolated specimen. It is not even necessary that the solid be piezoelectric to obtain these relationships; equations similar to (12) can be obtained for a permanently polarized sphere or for a permanently magnetized sphere [15]. For an ordinary dielectric which is electrically isolated, the equations hold trivially with $E = 0$, $D = 0$.

III. The Anelastic Loss Tangent

A. Derivation of Loss from General D-E Relation

In this section we develop an expression for the piezoelectric contribution to the anelastic loss tangent in a solid subjected to subresonant, sinusoidal loading. Suppose that, within the solid, the electric field and displacement are uniform, parallel, and related by the following:

$$D_3 = -E_3 e_0 \Lambda. \tag{13}$$

The quantity Λ will be left unspecified; however, note that for the thin plate, cylinder, and sphere, respectively, Λ is 0, 1, 2.

Let the solid obey the constitutive equations (5) and (6), i.e., let it exhibit dielectric and piezoelectric relaxation. The mechanical, dielectric, and piezoelectric loss tangents are defined as follows:

$$\tan\delta^{(s)}_{ijkl} = S''_{ijkl}/S'_{ijkl} \qquad \tan\delta^{(k)}_{ij} = k''_{ij}/k'_{ij}$$
$$\tan\delta^{(d)}_{ijk} = d''_{ijk}/d'_{ijk},$$

with no summation on the repeated indices. Combining (13) and (5), recalling the assumptions made earlier,

$$E_3 = \frac{-\sigma_{11} d^*_{311}}{e_0(\Lambda + k^*_{33})}. \tag{14}$$

Substituting this in (6), collecting terms, and simplifying, we obtain

$$\epsilon_{11} = \sigma_{11}\left\{\left[(S'_{1111})_E - \frac{(d'^2_{311} - d''^2_{311})(1 + \Lambda/k'_{33}) + 2d'_{311}d''_{311}\tan\delta^k_{33}}{e_0 k'_{33}\left((1+\Lambda/k'_{33})^2 + \tan^2\delta^k_{33}\right)}\right] - i\left[(S''_{1111})_E + \frac{(d'^2_{311} - d''^2_{311})\tan\delta^k_{33} - 2d'_{311}d''_{311}(1+\Lambda/k'_{33})}{e_0 k'_{33}\left((1+\Lambda/k'_{33})^2 + \tan^2\delta^k_{33}\right)}\right]\right\}. \tag{15}$$

This may be written as

$$\epsilon = \sigma\,\{S' - iS''\}, \tag{16}$$

in which the compliance in the brackets is what is measured in the specific geometry under consideration. This compliance is not the compliance at constant field which appears in the constitutive equation (6); for a particular specimen geometry, neither the field nor the electric displacement will be constant in general; they are determined by the geometry and therefore do not appear as independent variables in (15). These statements apply, of course, only to the situation considered here: dynamic mechanical loading with no external electrical perturbation.

The term subtracted from S' in (15) may be thought of as the piezoelectric contribution to the storage compliance: the presence of piezoelectric coupling reduces the compliance and causes the material to appear stiffer. If we consider a flat plate specimen, $\Lambda = 0$; and if there is no dielectric or piezoelectric relaxation, $\tan\delta^k_{33} = 0$, $d''_{311} = 0$. Since $\Lambda = 0$ implies that the electric displacement is zero, we obtain

$$S'^D_{1111} - S'^E_{1111} = -\frac{d^2_{311}}{e_0 k'_{33}},$$

which is equivalent to (3) in which the sum has collapsed into a single term as a result of the assumptions made in obtaining (15). The results, therefore, reduce to those of the classical theory in which no losses or piezoelectric phase angles occur.

The term added to S'' in (15) represents a piezoelectric contribution to the loss compliance. The presence of piezoelectricity and dielectric relaxation generally increases the mechanical energy loss. The piezoelectric contribution to the anelastic loss tangent, obtained by dividing the above term by S', is

$$\tan\delta^s_{1111} = \frac{(d'^2_{311} - d''^2_{311})\tan\delta^k_{33} - 2d'_{311}d''_{311}(1+\Lambda/k'_{33})}{e_0 k'_{33} S'_{1111}\left((1+\Lambda/k'_{33})^2 + \tan^2\delta^k_{33}\right)}. \tag{17}$$

This is compared in Table I with the results of other investigators; the comparison is discussed in Section IV. S' in this expression is the storage compliance for the geometry in question; this differs from the constant-field compliance $(S')_E$ which appears in the constitutive equation, as shown in (15). In cases of weak coupling the difference between these compliances is small.

The effect of the piezoelectric relaxation term d'' is much more pronounced in the contribution to the anelastic relaxation [(17) and the term containing $(S'')_E$ in (15)] than in the contribution to the storage compliance. This may explain why the classical theory of linear piezoelectricity [8], [9], [16] which addresses elastic effects in the absence of dissipation, is accurate for this type of problem (prediction of piezoelectric stiffening) despite the neglect of piezoelectric phase angles. As an example, consider a material for which the classical coupling coefficient in the thin plate geometry is $\xi^2 = 0.5$. Then the piezoelectric contribution to the storage compliance represents a factor two decrease in the compliance. If dielectric relaxation $\tan\delta^k = 0.03$ is introduced, then the piezoelectric contribution to the compliance is reduced by about 0.1 percent. If piezoelectric relaxation $\tan \delta^d = 0.03$ is introduced, then the piezoelectric contribution to the storage compliance is increased by 0.1 percent. Suppose now that the anelastic loss tangent at constant field is $(\tan \delta^S)_E = 0.03$. Then with "pure" piezoelectricity alone, the loss at constant displacement for an isolated thin plate is $(\tan \delta^S)_D = 0.06$, since the storage compliance in the denominator has been reduced by the piezoelectric reaction. If dielectric loss $\tan \delta^k = 0.03$ is again introduced, then $(\tan \delta^S)_D = 0.09$, a 50-percent increase over the case for zero dielectric loss. If piezoelectric relaxation, $\tan \delta^d = +0.03$ occurs, then we have $(\tan \delta^S)_D = 0.03$. The anelastic loss, therefore, is much more sensitive to variations in the dielectric and piezoelectric loss tangents than is the storage compliance.

B. Generalized Coupling Coefficient

The expression on the right in (17) can be decomposed into terms which contain expressions which resemble the piezoelectric coupling coefficient. Define for the thin plate $(\Lambda = 0)$, the following dimensionless quantities:

$$\kappa_0^2 = \frac{(d'^2 - d''^2)}{(S')_E e_0 k' (1 + \tan^2 \delta^k)} \tag{18}$$

and

$$\kappa_1^2 = \frac{2d'd''}{(S')_E e_0 k' (1 + \tan^2 \delta^k)}, \tag{19}$$

in which we note that $\mathrm{Re}\,[d^{*2}] = d'^2 - d''^2$ and $-\mathrm{Im}\,[d^{*2}] = 2d'd''$.

Then for the thin plate, we obtain from (15)

$$\frac{(S')_D - (S')_E}{(S')_E} = -\,[\kappa_0^2 + \kappa_1^2 \tan \delta^k] \tag{20}$$

$$\frac{(S'')_D - (S'')_E}{(S')_E} = +\,[\kappa_0^2 \tan \delta^k - \kappa_1^2]. \tag{21}$$

If the piezoelectric coupling is weak, so that $(S')_D \cong (S')_E$, then (21) represents the piezoelectric contribution to the loss tangents, see also (17). Even in the weak-coupling case, piezoelectric losses may account for most or all of the observed mechanical loss; for example, in photoconductive cadmium sulfide the background mechanical loss $(\tan \delta^S)_E$ is very small [17]. For strong coupling, the contribution to the storage compliance becomes appreciable, and (15) may be used directly to calculate the loss tangents. If we have $d'' << d'$, then $\kappa_1^2 = 2\kappa_0^2 \tan \delta^d$, a considerable simplification.

In the absence of all relaxation ($k'' = 0, d'' = 0, S''_E = 0$), κ_0^2 reduces to the coupling coefficient $\xi_{311}^2 = d_{311}^2/(S_{1111})_E\, e_0 k_{33}$ in the traditional theory [18], [19]; κ_1^2 vanishes. If dielectric relaxation occurs, κ_0^2 represents the ratio of the maximum energy extractable from the material in electrical form to the maximum energy input in mechanical form, as in the traditional theory. Since the dielectric coefficients depend on frequency, the rate of energy extraction must correspond to the frequency of mechanical excitation, if this agreement is to be obtained. If dielectric, piezoelectric, and anelastic relaxation all occur, this simple energetic interpretation of κ_0^2 is no longer valid. The coefficient κ_1^2, which is related to the out-of-phase piezoelectric coupling, can be either positive or negative depending on whether d'' is positive or negative. Since strain and electric field are nonconjugate variables, there is no theoretical energetic constraint on the sign of d'' as there is on the sign of k'' and S'' [20]; experimentally, both positive and negative d'' have been observed [20]. These considerations, as well as the discussion in Section III-A, cast doubt on the validity of an interpretation of d'' as an imperfection in the conversion of energy [5].

Both κ_0^2 and ξ_{311}^2 are bounded from above by unity, therefore the piezoelectric contribution to the anelastic loss tangent of a thin plate subjected to subresonant dynamic loading can be no greater than the dielectric loss tangent, provided that $d'' \geqslant 0$. The actual anelastic loss observed may be greater than the piezoelectric contribution, since a variety of loss mechanisms may contribute to the total loss.

The coupling coefficient, both in the relaxation-free traditional theory [19] and in the present analysis, will depend on the geometry and on the mode of vibration. For subresonant mechanical loading of a specimen with $\Lambda = 0$, 1, or 2, define Λ-dependent coefficients:

$$[\kappa_0(\Lambda)]^2 = \frac{(d'^2 - d''^2)}{(S')_E e_0 k' \left((1 + \Lambda/k')^2 + \tan^2 \delta^k\right)}$$

$$[\kappa_1(\Lambda)]^2 = \frac{2d'd''}{(S')_E e_0 k' \left((1 + \Lambda/k')^2 + \tan^2 \delta^k\right)},$$

which reduce to (18) and (19) respectively for $\Lambda = 0$. Then

$$\frac{S' - (S')_E}{(S')_E} = -\,\left[[\kappa_0(\Lambda)]^2 (1 + \Lambda/k') + [\kappa_1(\Lambda)]^2 \tan \delta^k\right]$$

$$\frac{S'' - (S'')_E}{(S')_E} = +\,\left[[\kappa_0(\Lambda)]^2 \tan \delta^k - [\kappa_1(\Lambda)]^2 (1 + \Lambda/k')\right].$$

Then as Λ increases, which represents increasing electric flux leakage out of the specimen, the piezoelectric contribution to the loss compliance decreases. The contribution to the storage compliance also decreases with increasing Λ, provided that the dielectric loss is very small; if the dielectric loss tangent is large, then the contribution to the storage compliance may first increase in magnitude, then decrease with increasing Λ.

IV. Discussion

A. Comparison with Other Results

The results obtained in the present study and results obtained by other investigators are compared in Table I, with salient assumptions made in each case. Kyame [21] and later Hutson and White [11] considered plane ultrasonic waves in an infinite domain of piezoelectric semiconductor, with a single dielectric relaxation time due to conductivity. The latter authors considered nonlinear terms associated with semiconductors which are not included in the equation in Table I. For a single relaxation time, the dielectric loss tangent in the present study reduces to $\tan \delta^{(K)} = \omega\tau/(1 + \omega^2\tau^2)$. Hutson and White's results should be compared with the present results for the thin plate, since in both cases there is no leakage of electric flux. The results differ as follows: 1) Hutson and White's results contain no effects due to piezoelectric relaxation d'', since d'' was not considered. 2) When converted to a form containing the piezoelectric d coefficients, these results contain the modulus C in the numerator, which differs from the inverse of the compliance found in the present results. This difference arises as a result of the constraint on Poisson contraction in the plane wave configuration. 3) The predicted complex modulus has the form $C' - iC''$ in [11, II.15]. This form entails energy gain, which does not occur in passive materials. The sign reversal apparently comes from the choice of a real dielectric constant and the associated boundary conditions.

In Ogawa's study [12], d'' was also neglected, but depolarization effects due to differences in specimen geometry were considered. The depolarization factor L, considered to be real, is related to the quantity Λ in the present study by $L = 1/(\Lambda + 1)\, e_0$. The quantity L is analogous to the demagnetization factor considered in magnetic studies. Ogawa treated $1/e_0$ (for a thin plate) and 0 (for a long rod). If the present results are specialized to the case of the thin plate, with no piezoelectric relaxation, and with a single relaxation time, then they are in agreement with Ogawa's results for $L = 1/e_0$. Ogawa modeled experimental results on photoconductive cadmium sulfide with his theory; by treating L as an adjustable parameter, he obtained qualitative agreement with experiment [17]. However, for large values of conductivity the predicted anelastic loss substantially exceeded the measured loss. One would expect the reverse to be true, since causes other than piezoelectric coupling can only add positive increments to the anelastic loss in a passive material. The presence of a finite d'', neglected by Ogawa, could account for this discrepancy. Ogawa also measured the losses in cadmium sulfide crystals of different shapes [17]. Long slender crystals exhibited smaller anelastic losses than plate-shaped ones, as predicted by Ogawa's theory [12] or by the present results. A quantitative comparison cannot easily be made in either case, since the specimen shapes used are not amenable to a straightforward computation of the depolarization factor L or of Λ.

Except for the factor π, the results of Yamaguchi and Takahashi [13] are equivalent to the present results, when the latter are specialized to the case of the thin plate, with no piezoelectric relaxation, and if the dielectric loss is small, i.e., $\tan^2 \delta^k << 1$. These authors also compared their theoretical expression with experimental data and observed that the calculated anelastic loss exceeded the measured loss by more than a factor two in materials with a relatively large loss. Again, this type of discrepancy is paradoxical given the fact that the material is passive. In this case, both the neglect of d'' and the neglect of depolarization effects (the specimens were actually long prisms, not thin plates) may have contributed to the discrepancy.

The expression obtained by Holland [6] for the piezoelectric contribution to the anelastic loss, contains d'' explicitly. A crucial assumption in the analysis leading to this expression is that the phase angle between stress and electric field may be set equal to zero since these quantities are independent variables. In the framework of the present study, if we eliminate D from (5) by means of one of the field-displacement relations (7), (11), or (12), we observe that the phase angle between stress and electric field does not in general vanish. It is conceivable that this phase angle could be made to vanish by simultaneously applying an appropriate stress and electrical signal; however, the stimulus would have to be different for each specimen geometry. It is not clear what the meaning of an "anelastic loss" measured this way would be.

As discussed earlier in Section II, it is conceptually reasonable to regard as free that charge not associated with dipole motion. Such a view is taken in treatments of lossless dielectrics [16]. However, when this assumption is made in the analysis of lossy piezoelectrics, a term containing k'' appears in the D-E relation for $\Lambda = 0$ (zero flux leakage), both in the study by Hutson and White [11] and in a preliminary report by the present author [14]. With this D-E relation and a constitutive equation with real D, a sign reversal is obtained which implies amplification rather than damping in the former case. This D-E relation and a constitutive equation with complex D gives rise to deviations from the true losses for large dielectric loss in the latter case. The preliminary results in [14] approximate the anelastic loss if all phase angles are small; the salient conclusions, i.e., that anelastic losses in piezoelectric media are geometry dependent and can be reduced by d'', are unchanged by the present study. The meaning of free charge in the boundary conditions is not apparent in the results of analyses of losses; this meaning is elucidated by certain gedanken experiments.

B. Significance

The expressions obtained above permit the calculation of the effective mechanical loss tangent and compliance of piezoelectric bodies with certain shapes, given the constant-field coefficients in the constitutive equations. Such expressions are expected to be useful in cases for which the performance of a given material, used in a variety of geometrical configurations, is to be predicted. The piezoelectric phase angles can have a significant effect on the results of calculations of this or similar nature. Neglect of these phase angles may account for some of the discrepancies which have appeared in the literature. For example, Desilets *et al.* [22] presented a design method, supported by careful experimentation, for efficient, broad-band, thin disk transducers. *Inter alia*, these authors observed a round-trip insertion loss of 6.5 dB at midband in a matched lead metaniobate transducer, compared with a predicted insertion loss of 1 dB; various experimental nonidealities were suggested as possible causes for the difference. In light of the present study, piezoelectric relaxation may be considered as a possible candidate for a cause for this type of discrepancy. In addition, both depolarization effects and piezoelectric relaxation as considered in the present study provide mechanisms whereby the paradoxical overestimates of mechanical losses in earlier treatments can be corrected.

The expressions obtained in the present study refer to a configuration in which the electrically isolated specimen is subjected to an oscillatory stress history at a frequency well below any resonance. Such a configuration may be used in studies of polymers [23] or in piezoelectric materials for which a sharp resonance cannot be obtained or for which data are to be obtained over several decades in the low frequency domain [3], [20]. The configuration approximates the behavior of a crystal which is used to receive ultrasonic waves at a frequency below any crystal resonances, if the amplifier attached to the crystal has a sufficiently high impedance. Although the results obtained here do not apply directly to cases in which electrical excitation is applied to the material, it is expected that the out-of-phase piezoelectric modulus d'' will affect the behavior of piezoelectric materials in those cases as well.

V. Conclusion

Expressions have been derived for the piezoelectric contribution to the mechanical loss of a solid, with specimen geometry and electrical boundary conditions considered explicitly. The presence of positive out-of-phase piezoelectric modulus has the effect of reducing the predicted loss; piezoelectric phase angles have much more effect on the loss than on the storage compliance of a solid. The mechanical loss of a piezoelectric body is found to be dependent on its shape.

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