# Structurally robust materials with ultra-low thermal expansion via designed microscale architectures

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## Background

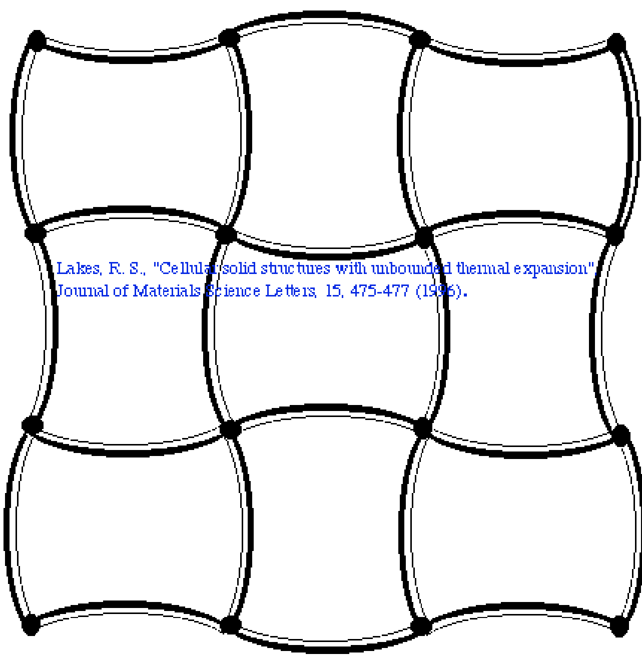
**The original idea.**

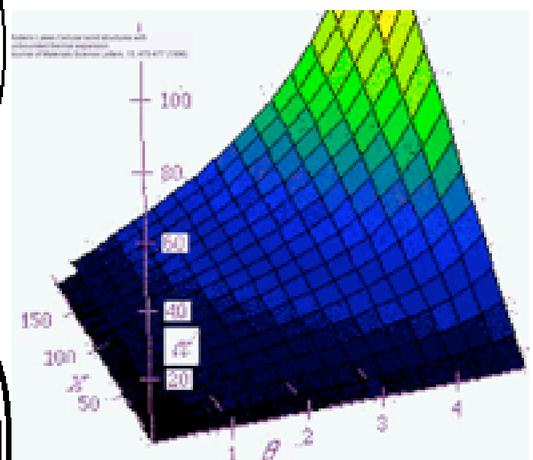
**Roderic Lakes Cellular solid structures with unbounded thermal expansion *Journal of Materials Science Letters,* 15, 475-477 (1996).**

We overcome the traditional bounds by relaxing assumptions used to derive them.

For arbitrary two-phase composite materials, bounds have been developed for the thermal expansion coefficient $\alpha$ of the composite in terms of that of each constituent; the upper bound is a rule of mixtures. In deriving these bounds it was assumed that there are only two phases, and there is no empty space in the composite.

In this article, it is shown that if one relaxes the assumption of no empty space, it is possible to generate cellular micro-structures which exhibit a thermal expansion coefficient much larger than that of either constituent. Zero expansion is also possible.




Roderic Lakes Cellular solid structures with unbounded thermal expansion
J. Materials Science Lett, 15, 475-477 (1996)

Left, a representative lattice. Right, expansion vs. rib angle and aspect ratio.

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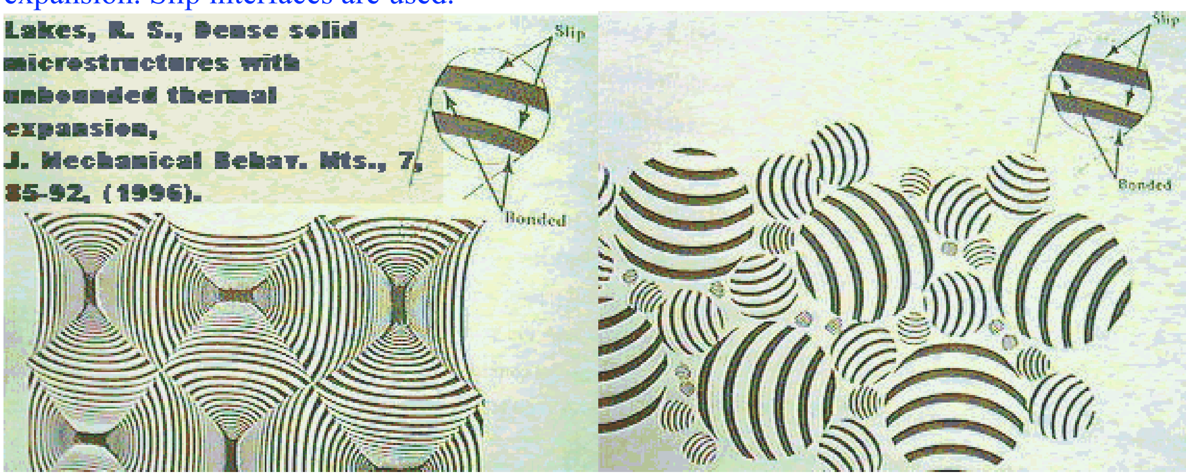
**Follow on.**

**Topology optimization**

If one has this idea, one can do topology optimization of the now included pore space as acknowledged by Sigmund, Torquato.

**Lattices vs. dense composites**

Do we need so much empty space? No: dense structures are possible with controlled expansion. Slip interfaces are used.



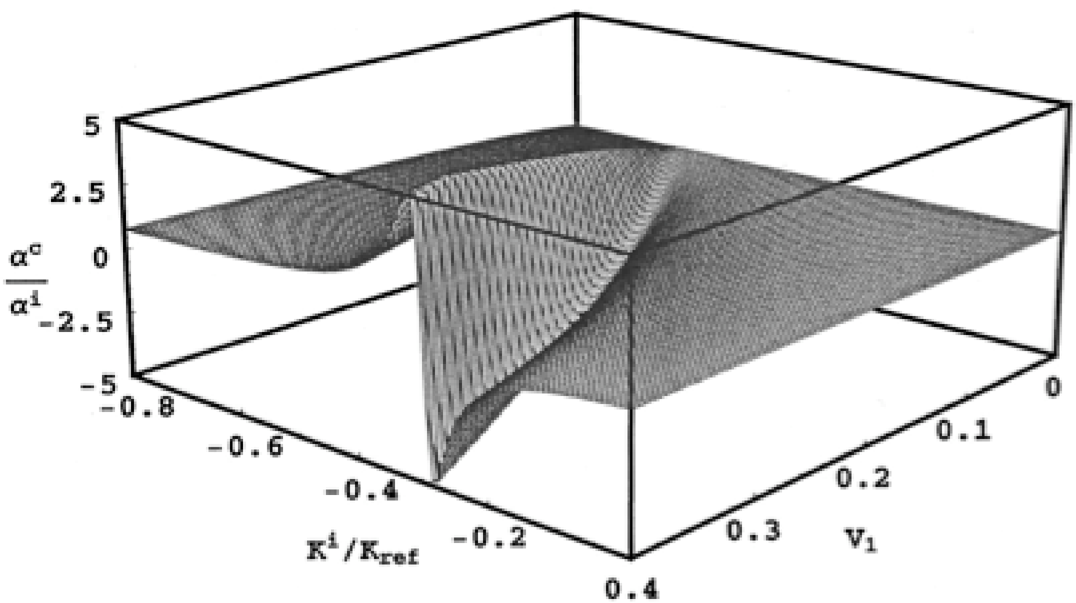

Random dense structure with laminated inclusions of different size. Curved laminae are alternately bonded and free to slip.

From **Lakes, R. S., Dense solid microstructures with unbounded thermal expansion, *J. Mechanical Behav. Mts.*, 7, 85-92, (1996).**

Use of negative stiffness; metastable constituents

We overcome the traditional bounds by relaxing a different assumption, that each constituent starts in a minimum energy state.

**Wang, Y. C. and Lakes, R. S., "Extreme thermal expansion, piezoelectricity, and other coupled field properties in composites with a negative stiffness phase",** ***Journal of Applied Physics*****, 90, 6458-6465, Dec. (2001).**




Wang, Y. C. and Lakes, R. S., "Extreme thermal expansion, piezoelectricity, and other coupled field properties in composites with a negative stiffness phase", *Journal of Applied Physics*, 90, 6458-6465, (2001).

The figure shows the thermal expansion coefficient (real part) of a Hashin – Shtrikman composite assuming a matrix phase of mechanical damping tan delta 0.05 in the bulk modulus with negative stiffness inclusions, as a function of inclusion stiffness and volume fraction.

Particulate composites with negative stiffness inclusions in a viscoelastic matrix are shown to have higher thermal expansion than that of either constituent and exceeding conventional bounds. It is also shown theoretically that other extreme linear coupled field properties including piezoelectricity and pyroelectricity occur in layer- and fiber-type piezoelectric composites, due to negative inclusion stiffness effects. The causal mechanism is a greater deformation in and near the inclusions than the composite as a whole.

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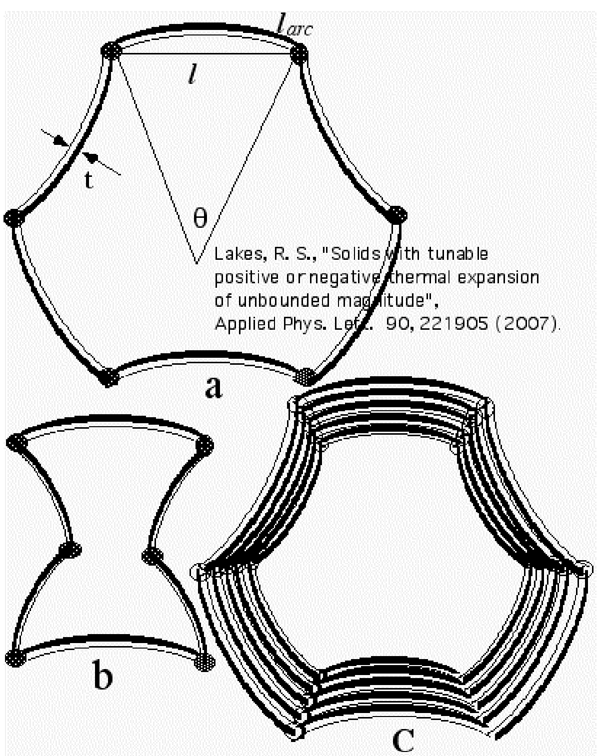
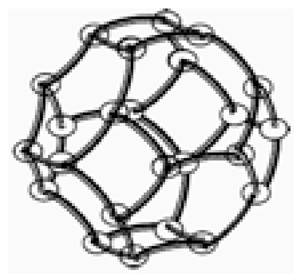
**Tuning via nesting and curvature**

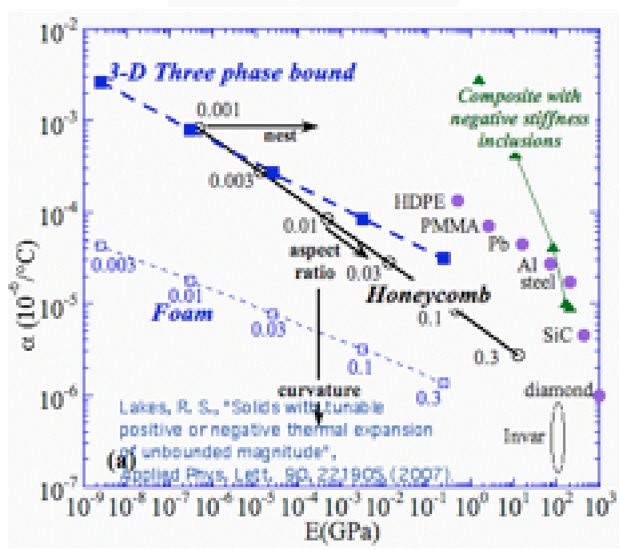
**Lakes, R. S., "Solids with tunable positive or negative thermal expansion of unbounded magnitude", *Applied Phys. Lett.* 90, 221905 (2007).**

Material microstructures are presented with a coefficient of thermal expansion $\alpha$ larger in magnitude than that of either constituent. Thermal expansion can be large positive, zero, or large negative. Three-dimensional lattices with void space exceed two-phase bounds but obey three-phase bounds; lattices and normal materials have a trend of expansion decreasing with modulus. Two-phase composites with a negative stiffness phase exceed bounds that assume positive strain energy density. For ribs of equal thickness and modulus,

$$\alpha = (\alpha_2 - \alpha_1)\frac{l_{\text{arc}}}{(h_1 + h_2)}\left[\frac{1}{2}\cot\frac{\theta}{2} - \frac{1}{\theta}\right]$$






Left. Hexagonal lattice cells with curved bimaterial ribs: a regular hexagonal cell, b inverted hexagonal cell for negative Poisson's ratio, and c nested lattice cell.

Right: map of expansion vs. modulus. Top right: foam cell

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**Recent progress**

- New graduate student Jeremy Lehmann has arrived following a summer co-op.

**Planning**

- Choice of constituent materials.
- Tuning of the angle: the correct curvature.
- Practical attainment of high modulus.